Presentation Title	**Understanding Primary Children's Thinking and Misconceptions in Decimal Numbers**
Format	**Paper Session [8.01]**
Sub-theme	**Assessment and Student Achievement**

Understanding Primary Children's Thinking and Misconceptions in Decimal Numbers

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Abstract

In Hong Kong, most of the primary school teachers are proud of their students in doing the four operations of decimal numbers. Inspecting a mathematics textbook might give the impression that, in order to be competent with decimal numbers, all that students need to do is remembering few rules for placing the decimal point, and otherwise perform operations with decimal numbers as if they were whole numbers. It seems to teachers and parents that primary students are able to line up decimal points and count decimal places. However, when asking a student to compare 4.8 and 4.75, he/she will choose 4.75 as the larger number because 75 is bigger that 8! In fact, the basic concepts of decimal number such as the place value and its relation with fraction are discussed at the early stage of learning decimals. However, the discussion is always too simplified and students do not understand thoroughly the underlying concept of decimal numbers but are pushed to learn the four operations of decimal numbers. Thus, students' computational skills on four operations of decimal numbers are merely rote learning and devoid of any meaning. With this reason, the research aimed at developing a tool and using it to uncover students' thinking and misconception on decimals. The significance of this research is to inform our pre-service student teachers and in-service teachers the general misconceptions of teaching and learning decimals, so that the focus and framework of teaching decimals could be restructured regarding students' learning difficulties which were revealed in this research.

Keywords: decimal numbers, fractions, four operations on decimals numbers

1. Introduction

Inspecting a mathematics textbook might give an impression that, in order to be competent with decimal numbers, all that students need to do is to remember a few mechanical rules for placing the decimal point, and otherwise perform operations with decimal numbers as if they were whole numbers (Steinle, 2004). However, when asking a student to compare 4.8 and 4.75, he/she will choose 4.75 as the larger number because 75 is bigger that 8 (Steinle, 2004). These observations reflect that the students have neither sense of the quantitative value of decimal numbers nor any understanding of the place value of each decimal place though the basic concepts of decimal numbers such as the place value and its relation with fraction are discussed at the early stage of learning decimals. Thus, students' computation skills on four operations of decimals are merely rote learning and devoid of any meaning. With this reason, the research aimed at firstly, unfolding students' misconception of decimal number and secondly, understanding whether the connections between conceptual and procedural knowledge exist when completing decimal tasks.

2. Mathematical analysis of decimal numbers

Decimal numbers are number that can be represented by base-10 numerals with digits to the right as well the left of the decimal point (Hiebert, 1992). The decimal notation system is designed to represent quantities that have been measured with units and parts of units. In contrast to common fractions, decimals require that the parts of units have a specified size relationship to the unit: tenth, hundredth, thousandth of a unit and so on. Hiebert (1992) concluded that decimal system could be captured in three principles. The following is the summary of his analysis.

Principle 1: The value of a digit is a function of its position in the numeral. The value of a particular position is determined by beginning with the unit and, if moving to the right, dividing the previous value by 10 and, if moving to the left, multiplying the previous value by 10. The ones position is marked with a decimal point on its immediate right. One consequence of this principle is that the relationship between any two adjacent places is that the one on the left is worth 10 times as much as the one on the right or, alternatively, the one on the right is worth one tenth as much as the one on the left.

Principle 2: The value of a digit is the product of its faces value and its places value. The face value is the value of a digit without considering its position in the numeral. Multiplying the face value of a digit times its place value gives the canonical value of the digit, the value in terms of the number of units of quantity. The value of a digit can be expressed in an endless variety of ways. For example, the canonical value of 6 could be 6 tenths or 60 hundredths.

Principle 3: The value of a numeral is the sum of the values of the individual digit.

3. Conceptual and procedural knowledge of decimals

Conceptual knowledge in general is defined as knowledge of those facts and properties of mathematics that are recognised as being related in some ways (Hiebert, 1986). Wearne and Hiebert (1988) hold similar view and describe it as semantic-based processes which mean connections between symbols with referents and development of rules by observing actions on referents. To be more specific, the conceptual knowledge of decimals is (i) knowledge of concrete or visual objects that can be measured by units, tenths of units, hundredths of units, and so on (Hiebert, 1992; Resnick el at., 1989); and (ii) knowledge of what happens when the decimals are moved, partitioned, combined, or acted upon in other ways (Hiebert, 1992). In short, a student who understands thoroughly the concept of decimals should be able to provide meaning for the symbol notation and provide the reasons for the symbols rules such as lining up the decimal points before adding or subtracting; dividing the denominator into the numerator to write a common fraction as a decimal (Hiebert, 1992).

In contrast, procedural knowledge is characterized by the absence of embedding relationships. Resnick and his colleagues (1989) hold similar view and state that children always construct erroneous rules without reference to the conceptual content or the meaning of arithmetic. Wearne and Hiebert (1988) describe it as syntactic processes which mean developing symbol-manipulation procedures and rountinizing the rules for symbols without thoroughly understanding the mathematical relationship behind. According to Hiebert (1986), procedural knowledge of decimals is best thought of in two parts. One part is the knowledge of written symbols in the syntactic system. This does not necessarily include knowledge of what the symbols mean, what quantities they represent (Hiebert, 1992). The second part is the set of rules and algorithms that are used to solve mathematics problems. These procedures are step-by-step prescriptions for moving from the problem statement to the solution.

Hiebert (1992) concludes that conceptual knowledge is the knowledge that is rich in relationships but not rich in techniques for completing tasks while procedural knowledge is rich in rules and strategies but not rich in relationships.

4. Methodology

A test on decimal numbers was constructed following the development and the measurement concepts in the Hong Kong primary mathematics curriculum (2000), which includes the following areas:

1. The meaning of decimal notation
2. The concept of place value in decimals
3. Comparison of decimals
4. The addition and subtraction of decimals
5. Renaming single unit to double units or vice versa
6. Multiplication of decimals
7. Division of decimals
8. Convert decimals into fractions

A content validity panel was set up, which includes primary mathematics education researchers, experienced primary mathematics teachers as well as pre-service primary mathematics education students. The test was modified after suggestions were given by the content validity panel. Pilot studies were done with a class of secondary one students twice, with a period of time in between, to test its reliability and it was satisfactory.

350 primary six students from six schools were invited to do the test. The test was administered to the participating students after the tested areas had been taught. Samples of participating students with observed critical features were interviewed. Their misconceptions are discussed in the next section and pseudo-names are used.

5. Findings and Discussion

Altogether thirty-three primary six students whose written tests were identified some critical features of misconceptions and seven students whose written test got nearly 98% correct were interviewed. It was purposefully to interview them for unfolding their understanding of conceptual knowledge in connection with procedural knowledge. They were probed to answer different questions on the first principle of decimal numbers, relationship between fraction and decimal, renaming a quantity from single unit to double units or vice versa, the concept of the rules for written computation procedures of four operations of decimal numbers such as lining up the decimal point for addition and subtraction. It was found that most of the students' knowledge was based on erroneous rules without reference to the conceptual content or the meaning of arithmetic. The following discussion will mainly focus on the seven more able students' interviews.

5.1 Case 1: Renaming single unit to double units or vice versa

One of the students, Peter, could tell clearly the procedure for renaming a quantity from double units to single unit but he did not conceptually make referent to fraction. The following is part of the interview (I: interviewer; S: student).

I: How can you change 1m and 32cm into 1.32m?

S: Teacher taught us that we could divide 32cm by 100. It is just that because of 100, you count two places from the most right of 32 to the left and place the decimal point there and put a zero on the left of the decimal point if it is 32 cm but it is 1m and 32cm, so you put 1 instead of zero.

I: Why do you do in this way? Why you count two places but not 3 or less places?

S: Because there are 100cm in 1 m and two zero in 100.

Only one of the students, David, mentioned one time the inter-relationship between fraction and decimal when explaining his procedures for renaming a quantity from double units to single unit. The following is what he said.

I: How do you rewrite 2.05m into 2m and 5cm?

S: The 2 in 2.05 means two metres and the 0.05 is 5/100 m which means 5 cm because there are 100 cm in one metre.

5.2 Case 2: The addition and subtraction of decimals

However, when further asking David the rationale for lining up decimal point for computing addition and subtraction and why it works, he could not explain conceptually that we have to put the same things together (i.e., units together, tenths together, hundredth together and so on) but apparently over-generalized certain aspects of whole number knowledge of operation symbol. The following interaction illustrates this point.

I: How can you get the answer of 11.24 – 3.07?

S: You firstly line up the decimal point. Then you treat them as whole numbers such that you subtract 307 from 1124.

I: How about 5.02 + 1.99?

S: Same as subtraction.

I: Why do you need to align the decimal point? How about if you do not align it?

S: I don't know. This is what my teacher told.

For some students especially the low ability students, they believed that if the decimal points were not aligned, then two decimal points would need to be placed in the answer and you did not know which place was for correct answer. The following typical interaction illustrates this point.

I: Why do you have to line up the decimal point?

S: If not, you would have two decimal points in two different places and you do not know which place is for correct answer.

I: Can both be correct?

S: No.

I: Why not?

S: They are different. Oh, I don't know.

Apparently, the students understood that this would be violate the syntactic conventions and consequently would be incorrect if the decimal points were not aligned. However, none of the students could tell the semantic meaning of the algorithmic procedure for addition and subtraction of decimal numbers.

<u>5.3 Case 3: The concept of place value in decimals</u>

Another simple syntax convention for whole number is that a zero to the right of a whole number increases the number by a factor of 10, whereas placing a zero on the left has no effect (Hiebert & Wearne, 1986). An opposite type of syntax applies to decimal numbers. Almost all the students were able to tell that the zero in 1.035 could not be taken away while that in 2.340 could. However, only one student, Peter, was able to tell the reason.

I: Can we take away the zero from 1.035 and 2.340?

S: The place value of zero in 1.035 is tenths. If you take away the zero, you change the number.

I: How about 2.340?

S: Yes, you can take away the zero as you don't change the number?

I: Why not?

S: It is at the end which means nothing.

Many students seem to not view zero as representations of quantities when it is at the end; however, they were unable to tell clearly the reason behind when zero was not at the end. The interaction below is a typical example.

S: If you take away zero from 1.035, you change the number?

I: What will be the number when you take away the zero? Bigger or smaller?

S: Bigger. Oh no, I don't know. I was just told by my teacher.

I: Then, how about 2.340?

S: The zero in 2.340 is not a number.

I: So?

S: You don't change the number if you take it away.

5.4 Case 4: Multiplication and division of decimals

When asking the students the rationale for algorithmic procedure for multiplication and division of decimal numbers, only one student, Roger, was able to make connection between fraction and decimal numbers, which is recorded as follows.

I: Why do you need to multiply the numbers by 10 before doing 9.5 ÷ 0.5 ?

S: To multiply the numbers is to expand the fraction to its equivalent form.

The student meant that you could rewrite 9.5 ÷ 0.5 into a fraction, $\frac{9.5}{0.5}$. To multiply both numbers was to make an equivalent fraction which was $\frac{95}{5}$ because 9.5 ÷ 0.5 could be best considered as 95 ÷ 5 and therefore, division of whole number could be applied. However, none of the students could tell conceptually the rationale for multiplication. The following is the typical answer from most of the students.

S: You have to firstly line up the most right of the decimal numbers. Then, you may just ignore the decimal points and apply the rule of multiplication for whole number. Finally, you have to count the decimal places in multiplicand and multiplier for putting the decimal point in the answer.

I: Does the rule work for all kind of multiplication of decimal numbers? Why?

S: I don't know. My teacher taught me that.

5.5 Case 5: Convert decimals into fractions or vice versa

Some of the students had difficulties in converting fraction into decimal numbers or vice versa. However, though some of them were able to do so, they could not tell the rationale for the algorithmic procedures. The following is an example from Raymond.

I: How can you convert $\frac{3}{20}$ into a decimal number?

S: I divide 3 by 20 (i.e, 3 ÷ 20)

I: How about $\frac{7}{25}$?

S: The same as before. You divide 7 by 25.

I: Why?

S: The teacher told that.

I: Why do you divide 3 by 20 and 7 by 25 then you can get the answers? What is the logic behind?

S: My teacher did not tell.

6. Conclusion and Further Research Directions

Understanding mathematics depends on the insight that connects similarities between related areas. Our analyses of performance on written tests indicated that the students are good at using mechanical algorithms to solve decimal problems; they can master the routine rules for symbol operations without thoroughly understanding the mathematical concepts. In other words, their procedural knowledge for the decimal number operations is excellent. However, our analyses and case study shows that their conceptual knowledge, for example the relationships between the decimal number system and the rational number system, is weak.

In a sense, of course, this result is not surprising, because it can be interpreted from an educational point of view as an instance of Hong Kong's examination oriented teaching and learning environment. However, what we concern are: i. "Does the current Hong Kong primary mathematics curriculum have any focus on decimal conceptual knowledge?" and ii. "Do the mathematics teachers know how to deliver decimal conceptual knowledge to the students?"

When we look at the Hong Kong primary mathematics curriculum more closely, we can find that conceptual connections between the decimal number system and the rational number system are missing. Perhaps, some additional materials can be written to fill up this missing gap. It is also likely that appropriate additional instruction could support the students to understand the connections between these two areas.

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