Hong Kong Mathematics Olympiad (2000 - 2001)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. a、b **和** c **分別為**ΔABC **的** $\angle A$、$\angle B$ **和**$\angle C$ **的相對邊的長度。若** $\angle C = 60^{\circ}$。**及** $\dfrac{a}{b+c}+\dfrac{b}{a+c}=P$，**求** P **的值。**

   a, b and c are the lengths of the opposite sides of $\angle A$, $\angle B$ and $\angle C$ of the ΔABC respectively. If $\angle C = 60^{\circ}$ and $\dfrac{a}{b+c}+\dfrac{b}{a+c}=P$, find the value of P.

2. **已知** $f(x) = x^2+ax+b$ **是** x^3+4x^2+5x+6 **和** $2x^3+7x^2+9x+10$ **的公因式。若** $f(P)=Q$，**求** Q **的值。**

   Given that $f(x) = x^2+ax+b$ is a common factor of x^3+4x^2+5x+6 and $2x^3+7x^2+9x+10$. If $f(P)=Q$, find the value of Q.

3. **已知** $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{Q}{a+b}$ **及** $\dfrac{a}{b}+\dfrac{b}{a}=R$，**求** R **的值。**

   Given that $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{Q}{a+b}$ and $\dfrac{a}{b}+\dfrac{b}{a}=R$, find the value of R.

4. **已知** $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ **及** $a^3+b^3=\text{S}$，**求** S。

Given that $\begin{cases} a+b=R \\ a^2+b^2=12 \end{cases}$ and $a^3+b^3=S$, find the value of S.

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]
Min. Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)

Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

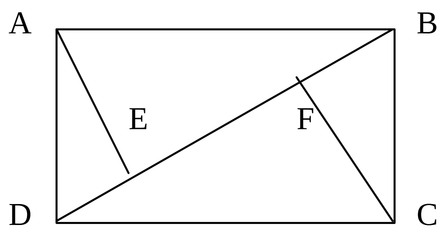
除非特別聲明，答案須用數字表達，並化至最簡。

1. **若** P **為整數，及** $5<P<20$。**若方程** $x^2-2(2P-3)x+4P^2-14P+8=0$ **的兩個根皆為整數，求** P **的值。**

   Suppose P is an integer and $5<P<20$. If the roots of the equation $x^2-2(2P-3)x+4P^2-14P+8=0$ are integers, find the value of P.

2. $ABCD$ **是一長方形。若** $AB = 3P+4$, $AD = 2P+6$ ， AE **和** CF **分別垂直於對角線** BD**，及** $EF = Q$**，求** Q **的值。**

   $ABCD$ is a rectangle. $AB = 3P+4$, $AD = 2P+6$. AE and CF are perpendiculars to the diagonal BD. If $EF = Q$, find the value of Q.



3. **某班學生的人數少於** $4Q$ **人。在一次數學測驗中有** $\frac{1}{3}$ **學生得甲等，** $\frac{1}{7}$ **學生得乙等，一半學生得丙等，餘下的學生都不及格。已知不及格的學生人數是** R**，求** R **的值。**

   There are less than $4Q$ students in a class. In a mathematics test, $\frac{1}{3}$ of the students got grade A, $\frac{1}{7}$ of the students got grade B, half of the students got grade C, and the rest failed. Given that R students failed in the mathematics test, find the value of R.

4. [a]**表示不大於** a **的最大整數。例如**[$2\frac{1}{3}$]=2。**已知方程**$[3x+R]=2x+\frac{3}{2}$**的所有根的和為** S**，求** S **的值。**

[a] represents the largest integer not greater than a. For example, [$2\frac{1}{3}$]=2. Given that the sum of the roots of the equation $[3x+R]=2x+\frac{3}{2}$ is S, find the value of S.

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐

+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐ ☐

Min. Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)

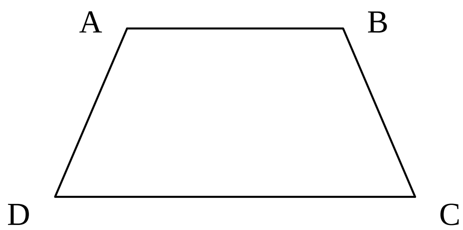
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. $ABCD$ **是一梯形，其** $\angle ADC = \angle BCD = 60^0$ **及** $AB = BC = AD = \frac{1}{2}CD$。**若把這梯形分割為** P **等份** $(P>1)$，**使其分割所得的每份與梯形** $ABCD$ **相似。求** P **的最小值。**

   $ABCD$ is a trapezium such that $\angle ADC = \angle BCD = 60^0$ and $AB = BC = AD = \frac{1}{2}CD$. If this trapezium is divided into P equal portions ($P>1$) and each portion is similar to trapezium $ABCD$ itself. Find the minimum value of P.



2. $(P+1)^{2001}$ **的個位數字與十位數字的和是** Q，**求** Q **的值。**

   The sum of the tens and unit digits of $(P+1)^{2001}$ is Q. Find the value of Q.

3. **若** $\sin 30^0 + \sin^2 30^0 + \ldots + \sin^Q 30^0 = 1 - \cos^R 45^0$，**求** R **的值。**

   If $\sin 30^0 + \sin^2 30^0 + \ldots + \sin^Q 30^0 = 1 - \cos^R 45^0$, find the value of R.

4. **設方程** $x^2 - 8x + (R+1) = 0$ **的根為** a **和** b。 **若** $\frac{1}{a^2}$ **和** $\frac{1}{b^2}$ **是方程** $225x^2 - Sx + 1 = 0$ **的根，求** S **的值。**

Let a and b be the roots of the equation $x^2 - 8x + (R+1) = 0$. If $\frac{1}{a^2}$ and $\frac{1}{b^2}$ are the roots of the equation $225x^2 - Sx + 1 = 0$, find the value of S.

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐

+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐ Min. ☐ Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. **已知** $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$**，** $x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$**，** $y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$。

   若 $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$**，求** P **的值。**

   Let $a^{\frac{2}{3}}+b^{\frac{2}{3}}=17\frac{1}{2}$, $x=a+3a^{\frac{1}{3}}b^{\frac{2}{3}}$ and $y=b+3a^{\frac{2}{3}}b^{\frac{1}{3}}$.

   If $P=(x+y)^{\frac{2}{3}}+(x-y)^{\frac{2}{3}}$, find the value of P.

2. **若一正** Q **邊形有** P **條對角線，求** Q **的值。**
   If a regular Q-sided polygon has P diagonals, find the value of Q.

3. **已知** $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$**，** $y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$。**若** $R=\frac{x^6+y^6}{40}$**，求** R **的值。**

   Let $x=\sqrt{\frac{Q}{2}+\sqrt{\frac{Q}{2}}}$ and $y=\sqrt{\frac{Q}{2}-\sqrt{\frac{Q}{2}}}$. If $R=\frac{x^6+y^6}{40}$, find the value of R.

4. **已知** $[a]$ **表示不大於** a **的最大整數。例如** [2.5] = 2。**若**

$S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$ **，求** S **的值。**

$[a]$ represents the largest integer not greater than a. For example, [2.5] = 2. If

$S=\left[\frac{2001}{R}\right]+\left[\frac{2001}{R^2}\right]+\left[\frac{2001}{R^3}\right]+\cdots$, find the value of S.

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐

+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐ Min. ☐ Sec.