Hong Kong Mathematics Olympiad (2000 - 2001)

Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. **已知** $(a+b+c)^2 = 3(a^2+b^2+c^2)$ **及** $a+b+c=12$。**求** a **的值**。
Given that $(a+b+c)^2 = 3(a^2+b^2+c^2)$ and $a+b+c=12$, find the value of a.

2. **已知**

$$b\left[\frac{1}{1\times 3}+\frac{1}{3\times 5}+\ldots+\frac{1}{1999\times 2001}\right]=2\times\left[\frac{1^2}{1\times 3}+\frac{2^2}{3\times 5}+\ldots+\frac{1000^2}{1999\times 2001}\right],$$

求 b **的值**。
Given that

$$b\left[\frac{1}{1\times 3}+\frac{1}{3\times 5}+\ldots+\frac{1}{1999\times 2001}\right]=2\times\left[\frac{1^2}{1\times 3}+\frac{2^2}{3\times 5}+\ldots+\frac{1000^2}{1999\times 2001}\right],$$ find

the value of b.

3. **一六位數** $1234xy$ **能同時被** 8 **和** 9 **整除。已知** $x+y=c$，**求** c **的值**。
A six-digit number $1234xy$ is divisible by both 8 and 9. Given that $x+y=c$, find the value of c.

4. **已知** $\log_x t = 6$，$\log_y t = 10$，$\log_z t = 15$。**若** $\log_{xyz} t = d$，**求** d **的值**。
Suppose $\log_x t = 6$, $\log_y t = 10$ and $\log_z t = 15$. If $\log_{xyz} t = d$, find the value of d.

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐ + Bonus score ☐ = Total score ☐

Team No. ☐

Time ☐☐ Min. Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)

Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. **已知** $x=\sqrt{7-4\sqrt{3}}$ **及** $\dfrac{x^2-4x+5}{x^2-4x+3}=a$**，求** a **的值。**

   Given that $x=\sqrt{7-4\sqrt{3}}$ and $\dfrac{x^2-4x+5}{x^2-4x+3}=a$, find the value of a.

2. E **是長方形** $ABCD$ **內一點。已知** EA**、**EB**、**EC **和** ED **的長度分別為** 2**、**$\sqrt{11}$**、** 4 **和** b**，求** b **的值。**

   E is an interior point of the rectangle $ABCD$. Given that the lengths of EA, EB, EC and ED are 2, $\sqrt{11}$, 4 and b respectively, find the value of b.

3. **已知** 111111222222 = $c\times(c+1)$**，求** c **的值。**

   Given that 111111222222 = $c\times(c+1)$, find the value of c.

4. **已知** $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ **及** $0< d <90$**，求** d **的值。**

   Given that $\cos 16^\circ = \sin 14^\circ + \sin d^\circ$ and $0< d <90$, find the value of d.

FOR OFFICIAL USE

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+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐☐ Min. Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)

Final Event 3 (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. **已知方程** $\sqrt{3x+1}+\sqrt{3x+6}=\sqrt{4x-2}+\sqrt{4x+3}$ **的解為** a**，求** a **的值。**

   Given that the solution of the equation $\sqrt{3x+1}+\sqrt{3x+6}=\sqrt{4x-2}+\sqrt{4x+3}$ is a, find the value of a.

2. **已知方程** $x^2y-x^2-3y-14=0$ **只得一組正整數解** (x_0, y_0)**。若** $x_0+y_0=b$**，求** b **的值。**

   Supoose the equation $x^2y-x^2-3y-14=0$ has only one positive integral solution (x_0, y_0). If $x_0+y_0=b$, find the value of b.

3. $ABCD$ **是一圓內接四邊形。**AC **和** BD **相交於** G**。已知** $AC=16$cm**,** $BC=CD=8$cm**,** $BG=x$cm **和** $GD=y$cm**。若** x **和** y **皆為整數且** $x+y=c$**，求** c **的值。**

   $ABCD$ is a cyclic quadrilateral. AC and BD intersect at G. Suppose $AC=16$cm, $BC=CD=8$cm, $BG=x$cm and $GD=y$cm. If x and y are integers and $x+y=c$, find the value of c.

4. **已知** $5^{\log 30}\times\left(\frac{1}{3}\right)^{\log 0.5}=d$**。求** d **的值。**

   Given that $5^{\log 30}\times\left(\frac{1}{3}\right)^{\log 0.5}=d$, find the value of d.

FOR OFFICIAL USE

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+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐☐ Min. Sec.

Hong Kong Mathematics Olympiad (2000 - 2001)

Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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1. x_1=2001。**當** n>1，$x_n = \dfrac{n}{x_{n-1}}$。**已知** $x_1x_2x_3...x_{10} = a$，**求** a **的值**。

   x_1=2001. When n>1，$x_n = \dfrac{n}{x_{n-1}}$. Given that $x_1x_2x_3...x_{10} = a$, find the value of a.

2. **已知** $1^3+2^3+3^3+\cdots+2001^3$ **的個位數字為** b，**求** b **的值**。

   Given that the unit digit of $1^3+2^3+3^3+\cdots+2001^3$ is b, find the value of b.

3. **甲乙兩人在一圓形跑道上同時同地相背以均速開跑。他們第一次相遇後，乙再跑** 1 **分鐘到達原起步點。已知甲和乙分別需要** 6 **分鐘和** c **分鐘繞跑道一周，求** c **的值**。

   A and B ran round a circular path with constant speeds. They started from the same place and at the same time in opposite directions. After their first meeting, B took 1 minute to go back to the starting place. If A and B need 6 minutes and c minutes respectively to complete one round of the path, find the value of c.

4. **方程** $x^2-45x+m=0$ **的兩個根皆為質數。已知兩根的平方和為** d，**求** d **的值**。

   The roots of the equation $x^2-45x+m=0$ are prime numbers. Given that the sum of the squares of the roots is d, find the value of d.

FOR OFFICIAL USE

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+ Bonus score ☐

Total score ☐

Team No. ☐

Time ☐☐ Min. Sec.