Keeping Probity in Probability:

Imprecise Probability or Impossible Probability?

Xiao-Li Meng

The transition from deterministic manipulation to probabilistic reasoning represented a monumental leap in scientific inquiry. Probability theory, by quantifying entities through distributions rather than mere numbers, provides a framework that is not only empirically relevant but also pairs epistemic integrity with theoretical coherence, as exemplified by Bayes' theorem. The emergence of this pairing, upon reflection, seems serendipitous. Imprecise Probability seeks to navigate beyond this horizon, addressing the challenge of incorporating ambiguous information that eludes precise probabilistic distribution. This endeavor is crucial when dealing with prior knowledge, often characterized more by its vagueness than its precision, without the need to concoct information to fit specific procedures, such as Bayes' updating.

However, the ambition to establish a unified generalization of Bayes' rule for imprecise probabilities encounters considerable obstacles, potentially insurmountable. The existing methodologies not only lead to conflicting inferences but also introduce phenomena like dilation, contraction, and sure loss—issues absent in the realm of precise probabilities and the conventional Bayes' rule. Through the lens of several well-known statistical paradoxes, we demonstrate that logical inconsistencies frequently stem from the aggregation of marginally plausible but collectively incompatible assumptions, reflecting the challenges mentioned earlier. We also explore the divergences and contradictions among the generalized Bayes' rule, Dempster's rule, and the Geometric rule as contenders for updating imprecise probabilities. These findings highlight a critical aspect of dealing with imprecise probability: the inevitable necessity to embrace a 'leap of faith', whether in selecting a prior distribution or an updating rule.

[This talk is based on the discussion article in *Statistical Science*: Gong and Meng (Statist. Sci. 36(2): 169-190 (May 2021). DOI: 10.1214/19-STS765) Judicious Judgment Meets Unsettling Updating: Dilation, Sure Loss, and Simpson's Paradox.]