Representations in problem solving in science: Directions for practice

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Abstract

In this paper we focus on some of the findings of the science education research community in the area of representations and problem solving. Problem solving depends on the construction and manipulation of mental models (internal representations) in the mind. A large knowledge base (declarative, procedural, strategic, situational, and schematic knowledge), working memory capacity, and metacognitive skills play an important role in the construction and manipulation of mental models, and therefore in problem solving. In this point, applications of research for classroom practice is considered. Finally, external representations are discussed. Using multiple representations when solving problems is beneficial for students, representational formats of problems affect student performance, and the utilization of representational learning strategies can lead to substantial improvements in problem solving.

Keywords: problem solving, representations, mental models, knowledge base, working memory, metacognitive skills.

Introduction

The HOCS (higher-order cognitive skills) capabilities of critical thinking, problem solving, and decision making are considered to be the most important learning outcomes that good teaching should aim for (Zoller, Lubezky, Nakhleh, Tessier, & Dori, 1995; Zoller, 2000). It is much-lamented that students often do not succeed in applying knowledge they acquire in lessons to everyday contexts. This seems to apply especially to science lessons (Friege & Lind, 2006). As a consequence, improving students' problem solving skills continues to be a major goal of science teachers and science education researchers. In order to achieve the ability to solve problems in science, there are two issues (Lee, Tang, Goh, & Chia, 2001): to develop in students problem solving skills through science education, and to look at the difficulties faced by students in this area and find ways to help them overcome these difficulties. Recently the types of knowledge needed to solve problems in science (Solaz-Portolés & Sanjosé 2007a) and an overview of research into cognitive variables that are involved in problem solving (Solaz-Portolés & Sanjosé, 2007b) have been reported. We now focus our attention on representations and how these representations affects problem solving.

Within the context of problem solving, it is useful to distinguish between internal and external representations. An internal representation is the way the problem solver stores internal components of the problem in his or her mind (Bodner & Domin, 2000). An external representation is something that stands for, symbolizes or represents objects and/or processes (Rosengrant, Van Heuleven, & Etkina, 2006). Examples in science include words, diagrams, equations, graphs, and sketches.

The purpose of this paper is fourfold: to present an overview of representations involved in problem solving in science; to show how representations mediate the performance of problem solvers; to emphasize relationships between internal representations of the problem (mental models) and some cognitive variables in problem solving; and to suggest some directions for classroom instruction to facilitate more effective problem solving.

Internal representations constructed during problem solving: mental models

According to the cognitive psychologist Mayer (1992) the process of solving problems has two steps, problem representation and problem solution. For problem representation, a learner needs to transform a problem's description to his or her internal mental representation in two stages: problem translation and integration. Problem translation extracts concepts from the textual description of the problem by using linguistic and semantic knowledge. Linguistic knowledge is used to comprehend the words' meanings in the textual description, while semantic knowledge means factual knowledge in the world. Problem integration requires a learner to connect sentences in a problems' description and produce a coherent representation. At this stage, schematic knowledge of problem classification is needed to integrate the pieces of information provided by the problem. Moreover, schematic knowledge allows a learner to determine the category of a problem. After the problem's description is translated into the learner's internal mental representation (mental model), it means that the learner has already comprehended the problem.

Pribul and Bodner (1987) concluded that the preliminary stages in the problem-solving process that involved disembedding the relevant information from the statement of the problem and restructuring or transforming the problem into one the individual understands are particularly important in determining the success or failure of the problem-solving process. Bodner and Domin (2000) suggest that an essential component of an individual's problem solving behaviour is the construction of a mental representation (mental model) of the problem that can contain elements of more than one representation system. The first representation establishes a context for understanding the statement of the problem. In some cases, this representation contains enough information to both provide a context for the problem and to generate a solution to the problem. In other cases, additional representations may be needed.

According to Slotta, Chi, and Joram (1995), problem solvers set up some initial representation based on key words in the problem statement. The information is often closely tied to real, familiar objects which in the case of the chemistry problems are images of laboratory apparatus or procedures. This representation is not linguistic but based on the individual's experience with, and knowledge about, the world. Bodner and Domin (2000) also found that successful problem solvers construct significantly more representations while solving a problem than those who are not successful. Unsuccessful problem solvers seem to construct initial representations that active an inappropriate schema (also referred to as frames or scripts, relate to one's knowledge about science) for the problem.

One of the most influential theories to be formulated in cognitive psychology in recent years is Johnson-Laird's (1983; 2000) theory of mental models. The theory seeks to provide a general explanation of human thought; at its core is the assertion that humans represent the world they are interacting with through mental models. In order to understand a real-world phenomenon a person has to hold, what Johnson-Laird describes as, a working model of the phenomenon in his or her mind. Johnson-Laird has formulated his mental model definition in his attempt to explain the reasoning processes in tasks of syllogisms and language comprehension. The author proposes that reasoning about a problem is facilitated if a person utilises a mental model that represents the relevant information in an appropriate fashion for the problem to be solved.

This theory is based on three main assumptions (Johnson-Laird, 2000).

- Each mental model represents a possibility. Models can represent relationships among three-dimensional entities or abstract entities; they can be static or kinematic. They underlie visual images, though many components of models are not visualizable.
- A mental model is iconic, that is, its parts correspond to the parts of what represents, and its structure corresponds to the structure of the possibility. The iconic nature of the model yields a conclusion over and above the propositions used in constructing the model.
- Mental models represents what is true according to the premises, but by default not what is false.

Johnson-Laird's mental model theory proposes a semantic, non-rule-based approach reasoning. According to mental model theory, human deduction depends on the construction and manipulation of analogical models in the mind. Model building and manipulation are processes that people carry out on line. Thus, models are not retrieved from long-term memory as rules or schemas are. To execute cognitive tasks, a person forms in working memory a mental representation, combining the information stored in long-term memory with the information on the task characteristics extracted by perceptual processes (Cañas, Antolí, & Quesada, 2001). Reasoning capacity limitations are explained within this theory as a consequence of the limitations in the human processing capacity. The limited capacity of working memory would restrict the number of possible models considered (Santamaría, García-Madruga, & Carretero, 1996). For this theory, the number of models is the main factor of difficulty in syllogistic reasoning. In fact, problems generating two or three mental models are more difficult than single-model problems (Johnson-Laird & Bara, 1984)

Mental models, problem solving, and cognitive variables: Directions for practice

The construction of a mental model results from links made between the elements of the problem description and the underlying knowledge base (Heyworth, 1998). Expert performance seems to lie in the organization of the experts' domain knowledge. Experts possess a large knowledge base that is organized into elaborate, integrated structures, whereas novices tend to possess less domain knowledge and a less coherent organization of it (Zajchowski & Martin, 1993). The way knowledge is organised allows optimised access to the long-term memory. The borders between long-term memory and working memory of experts become fluent so that the capacity of the working memory in comparison to a novices' memory is considerably expanded (Ericsson & Kintsch, 1995). Humans chunk content pieces together such that very large amount of content are concurrently accessible. Experts make use of big chunks that were developed over those years which they became experts (Brooks & Shell, 2006).

According to Kempa's studies (Kempa, 1991; Kempa and Nicholls, 1983) a direct connection emerges between cognitive structure (long-term memory structure) and problem-solving difficulties. These difficulties are usually attributable to one or more of the following factors.

- The absence of knowledge elements from a student’s memory structure.
- The existence, in the student’s memory structure, of wrong or inappropriate links and relationships between knowledge elements.
- The absence of essential links between knowledge elements in the student’s memory structure.
- The presence of false or irrelevant knowledge elements in the student’s memory structure.

The knowledge needed to solve problems in a complex domain is composed of many principles, examples, technical details, generalizations, heuristics, and other pieces of relevant information (Stevens & Palacio-Cayetano, 2003). The development of a knowledge base is important both in terms of its extent and its structural organisation. To be useful, students need to be able to access and apply this knowledge, but the knowledge must be there in the first place. Any claim that is not so, or that knowledge can always be found from others sources when it is needed, is naive (Dawson, 1993).

Shavelson, Ruiz-Primo and Wiley (2005) present a conceptual framework for characterizing science goals and student achievement that includes declarative knowledge (knowing that, domain-specific content: facts, definitions and descriptions), procedural knowledge (knowing how, production rules/sequences), schematic knowledge (knowing why, principles/schemes) and strategic knowledge (knowing when, where and how our knowledge applies, strategies/domain-specific heuristics). For each combination of knowledge type and characteristic (extent-how much?; structure –how it is organized?; and others), Li and Shavelson (2001) have begun to identify assessment methods. However, while we can conceptually distinguish knowledge types, in practice they are difficult to distinguish and assessment methods do not line up perfectly with knowledge types and characteristics.

Ferguson-Hessler and de Jong (1990) distinguished four major types of knowledge for the content of an adequate knowledge base with regard to its importance for problem solving.

- Situational knowledge is knowledge about situations as they typically appear in a particular domain. Knowledge of problem situations enables the solver to sift relevant features out of the problem statement.
- Declarative knowledge, also called conceptual knowledge, is static knowledge about facts and principles that apply within a certain domain.
- Procedural knowledge is a type of knowledge that contains actions or manipulations that are valid within a domain. Procedural knowledge exists alongside declarative knowledge in the memory of problem solvers.
- Strategic knowledge helps the student to organize the problem-solving process by showing the student which stages he should go through in order to reach a solution.

Working memory capacity plays an important role in many different types of problem solving (Welsh, Satterlee-Cartmell, & Stine, 1999). The ability to maintain information in a highly activated state via controlled attention may be important for integrating information from successive problem-solving steps, including the construction and manipulation of mental models. Working memory capacity may also be involved in a number of well-documented problem solving "difficulties" (Solaz-Portolés & Sanjosé, 2007c). Studies on the association between limited working memory capacity and information load in problem-solving provided support for the positive relationship between working memory and science achievement. Because working memory capacity limits the amount of information which can be concurrently processed, performance on science problem-solving tasks is expected to drop when the information load exceeds students' working memory capacity (Johnstone & El-Banna, 1986). Opdenacker, Fierens, Brabant, Sevenants, and Slootamekers (1990) study reported that students gradually decreased their chemistry problem-solving performances when the amount of information to be processed exceed their working memory capacity. This phenomena is also consistent with Sweller's (1994) cognitive overload theory, which posits that learning processes will be negatively affected if the cognitive load exceeds the limit of working memory capacity.

In science, mental capacity (M-space) is associated with students' ability to deal with problem-solving (Níaz, 1987; Tsaparlis, Kousathana & Níaz, 1998). Gathercole (2004) found a strong relationship between working memory capacity and science achievement: the correlation coefficients between working memory measure and science achievement ranged from 0.32 to 0.5. Danili and Reid (2004) found that students with high and low working memory capacity differed significantly in their performance on chemistry tests. Tsaparlis (2005) examined the correlation between working memory capacity and performance on chemistry problem-solving and the correlations ranged between 0.28 and 0.74.

From Anderson’s cognitive perspective, the components of science knowledge required to solve problems can be broadly grouped into factual (declarative), reasoning (procedural), and regulatory (metacognitive) knowledge/skills, and all play complementary roles (Anderson, 1980). According to O’Neil and Schacter (1999), to be a successful problem solver, one must know something (content knowledge), possess intellectual tricks (problem-solving strategies), be able to plan and monitor one’s progress towards solving the problem (metacognition), and be motivated to perform. Mayer (1998) examined the role of cognitive, metacognitive and motivational skills in problem solving, and concluded that all three kinds of skills are required for successful problem solving in academic settings.

Artz and Armour-Thomas (1992) suggest the importance of metacognitive processes in mathematical problem solving in a small-group setting. A continuous interplay of cognitive and metacognitive behaviours appears to be necessary for successful problem solving and maximum student involvement. Similarly, Teong (2003) demonstrated how explicit metacognitive training influences the mathematical word-problem solving. Results from his study revealed that experimental students outperformed control students on ability to solve word-problems. Experimental students developed the ability to ascertain when to make metacognitive decisions, and elicit better regulated metacognitive decisions than control students. Longo, Anderson and Wicht (2002) used an experimental design to test the efficacy of a new generation of knowledge representation and metacognitive learning strategies called visual thinking networking (VTN). In these strategies, students constructed network diagrams that contained words and figural elements connected by lines and other representations of linkages to represent knowledge relationships. Students who used the VTN strategies had a significantly higher mean gain score on the problem solving criterion test items than students who used the writing strategy for learning science (students used other strategies of learning including writing assignments). To get an overview of the characteristics of good and innovative problem-solving teaching strategies, Taconis, Fergusson-Hessler and Broekkamp (2001) performed an analysis of a number of articles published between 1985 and 1995 in high-standard international journals, describing experimental research into the effectiveness of a wide variety of teaching strategies for science problem solving. As for learning conditions, both providing the learners with guidelines and criteria they can use in judging their own problem-solving process and products, and providing immediate feedback were found to be important prerequisites for the acquisition of problem-solving skills. Abdullah (2006) indicated that there are only a few studies looking specifically into the role of metacognitive skills in physics despite the fact these skills appear to be relevant in problem solving. This researcher has investigated the patterns of physics problem-solving through the lens of metacognition.

Based on the overview on problem solving presented here, a number of instructional measures that will assist teachers are suggested below.

- A conceptual understanding of the topic must be obtained before students are given problems to solve, rather than trying to get this understanding by means of problem solving. A valuable science education will integrate the process of acquiring and applying conceptual knowledge. One technique that can be used by teachers to help students organise their understanding of a topic is concept mapping (Pendley, Bretz, & Novak, 1994). The introduction of a concept map can often assist students to understand the concepts and the relationships between them (Novak & Gowin, 1984).
- Instructional texts are dominated by declarative knowledge whereas procedural and situational knowledge is more implicit and has to be extracted, often by deep processing. Stimulating specific, deep study processes (e.g., explicating, relating, and confronting) might encourage students to change their learning habits (Ferguson-Hessler & de Jong, 1990).
- Traditional methods and instructional strategies of teaching science (lectures by the teacher, follow-the-recipe laboratory activities, exercise-solving recitation sessions, and examinations oriented toward algorithmic or lower-order cognitive skills) are not compatible with attaining conceptual learning and higher-order cognitive skills (Zoller et al., 1995). A major purpose of science education should be to develop instructional practices for developing scientific reasoning skills such as laboratory work, inquiry-based science, computer simulations, quantitative data analysis, constructing explanations, and critical thinking and decision-making capacity.

Improvement in reasoning skills has been shown to occur as a result of prolonged instruction and can lead to long-term gains in science achievement (Shayer & Adey, 1993). This study indicates that duration and intensity of exposure to reasoning situations are important factors for development of reasoning skills and that more individually targeted interventions may enrich/personalize the process.

- Encouraging an understanding of problems, rather than giving numerical procedures which may be memorized and used without understanding (Neto & Valente, 1997). This can be achieved using text-based or diagrammatic stimuli that require a knowledge of underlying concepts or basic theories of science. Qualitative discussions could be carried out while problems are solved on the chalkboard and also by getting students to work together while solving problems with students being asked to derive general procedures rather than mathematical solutions.
- Provide students with diverse, continual and prolonged problem-solving experiences. Associated with all problems are three variables: the data provided, the method to be used and the goal to be reached (Johnstone, 1993). Once students have derived and understood procedures for basic problems (recall of algorithms), they should be given plenty of practice to the other problem types, for example, problems unfamiliar to the student that require, for their solution, more than conceptual knowledge application, analysis, and synthesis capabilities, as well as making connections and evaluative thinking on the part of the solver. Give practice of similar problem solving strategies across multiple contexts to encourage generalization.
- Offer strategies in metacognition, such as teaching the existence of functional knowledge types and the role of problem schemata. Use problem-solving heuristics and metacognitive activities (Lorenzo, 2006). Explain the role of metacognitive skills in the steps in problem-solving. Metacognitive skills can be found in the steps of planning, reflecting (monitoring progress), checking (verifying results), and interpreting problem-solving (Abdullah, 2006).
- Alloway (2006) suggests that the learning progress of students with poor working memory skills can be improved dramatically by reducing working memory demands in the classroom. She recommends a number of ways to minimise the memory-related failures in learning activities: by using the instructions that are as brief and simple as possible, by reducing the linguistic complexity of sentences, by breaking down the tasks into separate steps, by

providing memory support, by developing in the students effective strategies for coping with situations in which they experience working memory failures, etc.

- It is useful for the teacher to understand that the M-demand (mental demand) of an item (problem) can be changed without changing its logical structure. This can facilitate student success by decreasing the amount of information required for processing and, avoiding working memory overload (Níaz, 1987). Johnstone, Hogg, and Ziane (1993) suggest physics problems can be presented in a way that reduces the noise input of the processing system, and consequently allows greater success for all students, particularly field-dependent students (students with worse ability to disembed information in a variety of complex and potentially misleading instructional context). This means words, combined with a diagram, can reduce memory overload.
- Sweller, van Merrienboer, and Paas (1998) argued that students only had to maintain the problem state and any problem-solving step for that state when solving goal free problems, thus reduced cognitive load.

External representations to facilitate problem solving

Using external representations through symbols and objects to illustrate a learner's knowledge and the structure of that knowledge can facilitate complex cognitive processing during problem-solving (Vekiri, 2002). Such external representations can help a learner elaborate the problem statement, transform its ambiguous status to an explicit condition, constrain unnecessary cognitive work, and create possible solutions (Scaife & Rogers, 1996). Larkin (1989) argued that an external representation supports human problem-solving by reducing the complexity of problem and its associated mental workload. Moreover, Bauer and Johnson-Laird (1993) showed that diagrams helped learners solve a problem more effectively and efficiently.

Learners have a limited working memory, and instructional representations should be designed with the goal of reducing unnecessary cognitive load. However, prior knowledge can determine the ease with which learners can perceive and interpret visual representations in working memory (Cook, 2006). Three issues developed from using multiple representations in problem solving: how students use multiple representations when solving problems, how different representational formats affect student performance in problem solving, and how the utilization of representational learning strategies can lead to substantial improvements in problem-solving.

Physics education literature indicates that using multiple representations is beneficial for student understanding of physics ideas and for problem solving (Dufresne, Gerace, & Leonard, 1997; Larkin, 1985; Van Heuvelen, 1991). These representations can include but are not limited to words, diagrams, equations, graphs, and sketches. However, there is less research on thought processes that students use while applying multiple representations in problem solving. The hypothesis of Rosengrant, Van Heuvelen and Etkina (2006) is that students are probably aware intuitively that they do not have the mental capacity to remember all the information in the problem statement, and thus use the representations to visualize an abstract problem situation. Their previous research (Rosengrant, Van Heuvelen, & Etkina, 2005) showed that students improve their chances of solving a problem correctly if they include concrete diagrammatic representations as part of the solving process.

Kohl and Finkelstein (2005) examined student performance on homework problems given in four different representational formats (mathematical, pictorial, graphical, and verbal), with problem statements as close to isomorphic as possible. They found that there were statistically significant performance differences between different representations of nearly isomorphic statements of problems. They also found that allowing students to choose which representational format they use improves student performance under some circumstances and degrades it on others. In another work (Kohl & Finkelstein, 2006a) reported that students who learnt physics using lots of representations were less affected by the specific representational format of the problem. Finally, these authors investigated in more detail how student problem-solving performance varies with representation (Kohl & Finkelstein, 2006b). They discovered that student strategy often varies with representation, and that students who show more strategy variation tend to perform more poorly. They also verified that student performance depends sensitively on the particular combination of representation, topic, and student prior knowledge.

Longo, Anderson and Wicht (2002) used knowledge representation and metacognitive learning strategies called visual thinking networking. In these strategies students constructed network diagrams which contained words and figural elements connected by lines and other representations of linkages to represent knowledge relationships.

Earth science learning was improved in the area of problem solving for students who used visual thinking networking strategies. Chan and Black (2006) investigated what learners need for constructing mental models to understand and reason about systems and scientific phenomena which can be described in text, pictures, and animation. Their results corroborated that, for simple and moderately systems, students did not perform significantly different on learning activities. However, as the systems became more complicated, students who directly manipulated the animation outperformed those in text-only groups and texts-and-static-visuals groups on the outcome measures. Mayer's (1999) research pinpointed some conditions under which multimedia learning can lead to substantial improvements in problem-solving transfer. Overall, students make better sense of a scientific explanation when they hold relevant visual and verbal representations in their working memory simultaneously. When multimedia messages are designed in ways that overload visual or verbal working memory, transfer performance is adversely affected.

Conclusions

According to the mental model theory, problem solving depends on the construction and manipulation of mental models (internal representations) in the mind. The construction of a mental model results from links made between the elements of the problem description and the underlying knowledge base. This knowledge base is composed of several types of knowledge: declarative, procedural, strategic, situational, and schematic knowledge. Working memory plays an important role in the construction and manipulation of mental models. Studies involving limited working memory capacity and information load support the positive relationship between working memory and problem solving. Moreover, the importance of metacognitive processes during problem solving have been advocated by many researchers. Based on the discussion, directions for the improvement of science problem solving skills can be suggested. These will include the key role of a large knowledge base, to decrease the information load in problem solving, and to offer measures in the field of metacognition.

For external representations it has been shown that using multiple representations when solving problems may be beneficial because representational formats can affect student performance and the use of representational learning strategies can lead to substantial improvements in problem solving.

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