A survey on the teaching of relative velocity and pupils' learning difficulties

Tin-Lam TOH

National Institute of Education
Republic of SINGAPORE
E-mail: tinlam.toh@nie.edu.sg

Received 31 Oct., 2006
Revised 8 Dec., 2006

Contents

Abstract

It has been five years since the chapter on relative velocity was first introduced into the Singapore Additional Mathematics curriculum. This paper reports some general finding on the teaching of relative velocity in mathematics classrooms and the pupils' learning difficulties on relative velocity. Some implications to the teaching of this topic are also discussed.

1.Introduction

Relative velocity was first introduced into the Singapore Additional mathematics curriculum in 2001. The inclusion of this chapter under the topic of vectors (see Ministry of Education, 2000) probably aims to build this chapter on the pupils' knowledge of vectors and is a special application of vectors in real life. As it is a mathematics topic, not so much on a thorough understanding of the physics concepts of relative velocity per se is expected from the students.

A quick survey of the recent typical questions on relative velocity from the Additional Mathematics examination papers show that, in fact, solving such problems requires a great deal of sound concrete knowledge of vectors and kinematics. For example, a student needs to know that when a particle travels with a constant velocity, there is not resultant force acting on the particle; it is a common misconception that a force causes motion (or the velocity) of the moving body. To put it diagrammatically, the hierarchy of learning for relative velocity can be summarised in the following diagram.

In order for learning to be effective, according to Gagne (1970), tasks of the lower order learning hierarchy must be acquired first before the tasks at the higher level. Thus, any learning difficulties of the pupils can either be the difficulties involving concepts of relative velocity and their applications or the tasks of the lower hierarchy, e.g. lack of content mastery of concepts of vectors, trigonometry or geometry.

It is the interest of the author to find out the pupils' general learning difficulties in relative velocity after it has been in the syllabus for five years.

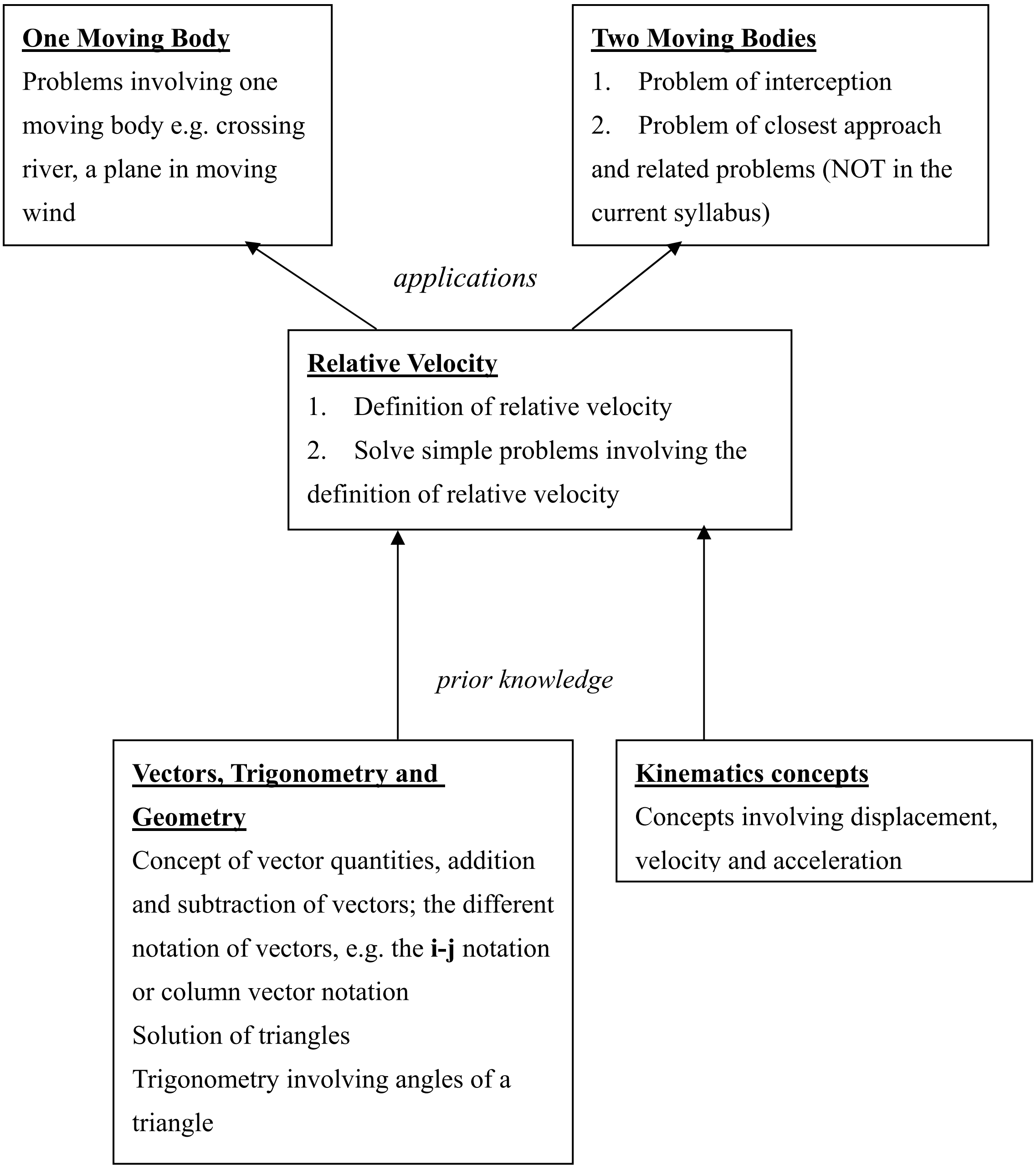

Figure 1: The learning hierarchy of relative velocity

2. Background

2.1 Brief description of the mathematics teachers' background

First we present a brief outline of the background of the mathematics teachers' content knowledge. The mathematics teachers can be classified under three categories: (1) teachers who majored or minored in mathematics in their undergraduate days; (2) teachers who were from engineering courses; and (3) teachers who were not from mathematics or engineering background.

With reference to the mathematics teachers who majored in mathematics in their undergraduate days : as they were taking university mathematics courses, many of them did not have contact with concepts in mechanics, relative velocity in particular, especially if their second teaching subject is not physics.

The second group of mathematics teachers, who were from engineering background, might have sufficient exposure with mechanics in their engineering courses. However, a talk with a few of my teacher trainees in National Institute of Education showed that they were not exposed much to the concepts involving kinematics leading to relative velocity.

The third group of mathematics teachers, who were from non-mathematics and non-engineering background, had virtually no experience with mechanics concepts in their undergraduate days.

It is no wonder that many mathematics teachers in schools might have difficulties with kinematics concepts and relative velocity. This was supported by the survey on the inservice teachers' misconceptions in kinematics (see Toh, 2005)

2.2 Teachers' Pedagogical Content Knowledge

Current research has shown that what teachers know has a direct impact on the quality of students learning (see for example Muijs and Reynolds, 2000; Wenglinsky, 2000; Ling et al, 2006). Teachers must be able to anticipate the possible errors and misconceptions that their students might have for each of the different topics in the syllabus (for example, Chua & Wood, 2005). Only by the anticipating of the pupils' possible misconceptions can teachers be able "to use a number of different strategies and how to coordinate between strategies depending on the teaching context" (Abd Rahman N, 2004). It is also equally important that the teachers' own misconceptions of the various topics in the syllabus (for example, see Toh, 2005 on misconceptions in kinematics) must be rectified.

It is recognized that in order to teach well, a mathematics teacher must know a great deal of mathematics (Usiskin, 2001), on top of having a thorough mastery of the content knowledge of the material in the existing mathematics curriculum. The types of mathematics knowledge that a mathematics teacher needs to know are discussed in detail with examples by Usiskin (2001).

In specific to relative velocity, a mathematics teacher should have a good grasp of the following subtopics (Toh, 2004)

1. Concepts in vectors and kinematics (including familiarity of using i-j notation and column vector notation for displacement, velocity and acceleration)
2. Basic concept and definition involving relative velocity
3. On crossing a river (involving one moving object)
4. Concept of relative velocity and the replacement displacement of two moving objects
5. Relative velocity and the problems of interception of two moving objects
6. Relative velocity and the problems of closest approach.

According to Usiskin (2001), a teacher needs to know more content knowledge than their pupils; this knowledge includes *generalization and extension* from what is required in the syllabus. Thus, even though the problem of closest approach (item 6 above) is not required in the O-Level curriculum, teachers are expected to have a sound content knowledge on this (Toh, 2004).

2.3 Pupils' Difficulties in Learning Science related concepts

According to Driver (1983), pupils' interpretation of science concepts is often at odds with the scientists' views. Pupils' preconceived ideas influence them to interpret new learning experiences, and it is exactly these preconceived ideas serve as barrier for their future progress. To quote Driver (1983) exactly:

> "...... all its experiences of pushing, pulling, lifting, throwing, feeling and seeing things stimulate the ability to make predictions about a progressively wider range of experiences. By the time .... it has already constructed a set of beliefs about a range of natural phenomena. In many cases, these beliefs or intuitions are strongly held and may differ from the accepted theories which science teaching aims to communicate."

Thus, effective teaching should involve teaching that entails teaching strategies that take into account these pupils' misconceptions due to preconceived ideas. These strategies should also emphasize conceptual understanding instead of rote learning (Abd Rahman N, 2004).

3. Method

As the objective of this study was to obtain information on the general teaching of relative velocity in the mathematics curriculum and the pupils' learning difficulties in this topic, a qualitative survey was carried out through a series of emails with seven teachers from five different secondary schools. Due to constraint of time and financial resources as this is not a funded research project, formal interviews were not conducted and replaced by a series of emails. As the sample size was small due to constraints of realities, it was not mathematically sound to perform inferential statistical analysis; only qualitative information was obtained.

In the first email with the participating teachers, they were asked to write down at least three of the problems that pupils normally encounter when learning relative velocity. The teachers' responses to the learning difficulties on relative velocity were recorded. The second and subsequent emails were sent to the responding teachers.

In the subsequent emails, attempt was also used to find out on the following three items of the general teaching of relative velocity. The rationales why these areas were selected are discussed below each item.

1. the teaching materials used;

Rationale: I would like to see if the existing material from the textbook is sufficient for their teaching purposes.

2. whether scale drawing was taught or allowed in solving problems on relative velocity; and

Rationale: If the pupils' learning difficulties were from the more basic hierarchy part (for example, difficulties lying in trigonometry or geometry and not direct relative velocity concepts), then this difficulty could have been reduced by allowing pupils to use scale drawing to solve problems on relative velocity.

3. whether any form of information technology, such Geometer's to illustrate concepts of relative velocity was used.

Rationale: Some difficult concepts of relative velocity can be well understood by using technological illustrations, e.g. the angle for a boat to steer crossing a river with current flowing downstream to have the minimum displacement downstream (see Toh, 2003).

However, not all teachers responded the subsequent emails. We present the information based on all the feedback given in the subsequent emails. In section 4 below, we present the pupils' learning difficulties in relative velocity; in section 5, we present some common phenomena on the teaching of relative velocity in schools.

4. Pupils' Learning Difficulties in Relative Velocity

From the feedback obtained, the pupils' learning difficulties in relative velocity can be classified as the following categories:

4.1. Psychological Factor – Rosenthal Effect

This first factor raised by two of the heads of mathematics departments turns out as a little surprise to the author. Rosenthal effect is a form of self-fulfilling prophecy or experimenter bias in a social setting suggested by the psychologist Robert Rosenthal (1976).

According to these two heads of departments, their teachers have indicated that relative velocity is a challenging (or difficult) topic to the teachers themselves; this feeling towards the topic could have turned into nonverbal cues in their classrooms and consequently their pupils felt that this topic is difficult and hence were unwilling to spend extra time on this topic and at the same time did not heed the teachers' advice on how to handle such questions.

Rosenthal effect could have been universal for all the topics in the syllabus; because the teachers personally find this topic difficult themselves, Rosenthal effect could have played a significant role in teaching relative velocity in contributing to the pupils' learning difficulty in relative velocity.

Also, one of the heads of department further pointed out that the teachers might have instilled in the pupils that giving up this topic on relative velocity does not affect one's chance of scoring good grades in Additional Mathematics provided one masters the other topics sufficiently well. This could also have affected pupils' learning in this topic.

4.2. Language of Relative Velocity

Many pupils have difficulty in interpreting the language of vectors, for example, the wind movement.

Example 1

> As an illustration given by one of the teachers, many pupils cannot distinguish between the case when "the wind is blowing along north" as contrasted to "the wind blowing from the north".

Example 2

> According to another teacher, many pupils and several teachers had difficulties in interpreting the direction of rain in the following information: "A man is walking along a horizontal road at 1.2 m/s. The rain is coming towards him and appears to be falling with a speed of 4 m/s in the direction which makes an angle of 60° with the horizontal......."

4.3. Related Science concepts

Most pupils who are not offering pure physics as another main subject have difficulty in understanding the technical terms involved in relative velocity. These concepts raised by the participating teachers are discussed below.

4.3.1 Velocity and Displacement

When both velocity and displacement, being vector quantities, can be represented in the standard **i-j** notation or the column vector forms, there is a great deal of confusion among students between velocity vectors and displacement vectors. Consequently the pupils are not able to use the information in solving the word problems.

Example 1

In a word problem when both displacements and velocities are given in **i-j** notations, the pupils have difficulties in interpreting the information.

4.3.2 Physical Meaning of Relative Velocity

It is a common problem raised by the teacher participants and the students that there is a great deal of difficulty in distinguishing true velocity and relative velocity. Consequently, it becomes a rather difficult task for the pupils to translate the information into drawing a correct vector diagram.

Furthermore, it is difficult for pupils to visualize (and for teachers to explain) the concepts related to relative velocity, for example, the velocity of one object relative to another; the velocity of airplane in still air.

4.4 Difficulties related to Mathematics

While the above problems tend to occur across most pupils learning the chapter on relative velocity, the weaker pupils also have additional difficulties involving mathematics related items. Some of the problems that were mentioned by the teacher participants are listed below:

i. the use of angles formed by non-parallel lines, especially when the line segments (drawn) to represent the lines do not appear to intersect;

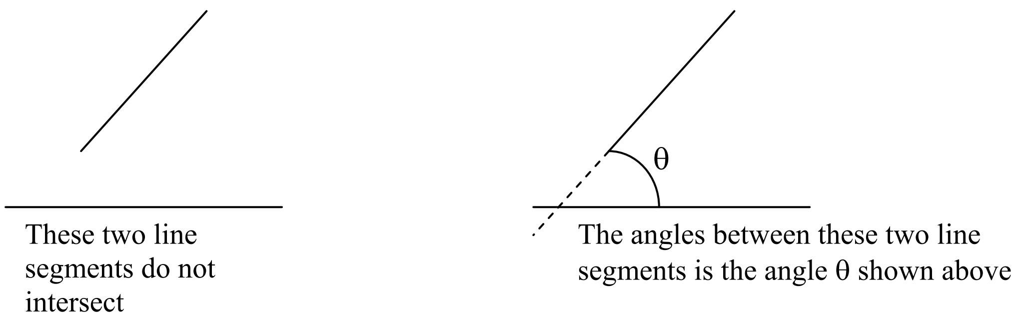

Figure 2: Students' difficulties with angles between two nonintersecting line segments

ii. the conversion between vectors in **i-j** form and column vector form and the related concepts of vectors in kinematics form (e.g. the magnitude of the velocity vector gives the speed; the magnitude of the displacement vector yields the distance from the origin O);

iii. the use of bearings of locating positions;

iv. performing vector addition and subtraction by using vector diagrams; and

v. the use of sine and cosine rules in solving triangles.

5. Survey on Teaching of Relative Velocity

5.1. Teaching materials used

It was generally felt among the teachers that the current textbooks do not provide sufficient related materials on relative velocity. The teachers prepare their own numerical examples for use in the classrooms. These self-prepared examples are modifications of the existing pool of O-Level questions and the similar questions used by other schools for their own internal tests and examinations.

5.2. Use of Scale Drawing in Solving Problems involving Relative Velocity

From the response, the participants mentioned that the teachers were aware that scale drawing is allowed for examination purposes. However, the teachers might not have highlighted the use of scale drawing as one alternative approach to solving problems on relative velocity. Consequently, according to two of the participants who were the heads of the mathematics departments of their schools, practically no students used scale drawing in their school tests and examinations.

5.3. Use of Information Technology (IT)

Two teachers responded that since the main problem of relative velocity is more "back to basics". When probed further into the definition of "back to basics", one participant replied she means "to be able to solve the typical examination type problems". In this respect, the use of information technology (IT), which is more of illustrating the relative velocity concepts, is not too crucial compared to solving examination type problems.

6. Some Implications for Teaching of Relative Velocity

Rosenthal Effect was being mentioned in the preceding paragraphs. It is thus important that the teachers orientate their pupils in the correct frame of mind before teaching this topic, as a teacher in class is the role model and should avoid instilling in the pupils the idea that this topic is difficult for most pupils or unimportant for examination purposes if he wants to ensure pupils learn this particular topic well.

Another problem that was highlighted was that many of these concepts are science related. Thus, some form of interdisciplinary effort could be made between the mathematics and science departments in aligning the science and mathematics subjects in the secondary schools, especially in delivering the concepts involving kinematics, especially displacements and velocities.

It is interesting to note that scale drawing is practically not emphasized in learning relative velocity. The use of scale drawing in solving problems involving relative velocity helps pupils save significant effort in trigonometric and geometric computations and is particularly useful for students who are weaker in geometry and trigonometry.

The use of IT can be used to help pupils visualize related concepts on relative velocity, which is going back to the basics of this chapter in the sense of enhancing pupils' understanding of mathematical concepts. Many concepts, which otherwise need tedious mathematical computations, can be illustrated and visualized almost intuitively with the use of IT (Toh, 2003). Thus, in this sense, the use of IT can serve more as getting the basics right than as enrichment activity.

7. Conclusion

The introduction of relative velocity into the Additional Mathematics curriculum serves as an application of vectors in real life, and a reinforcement of other more elementary mathematics topics. Teachers should take into account the learning difficulties of the weaker mathematics students. Furthermore, as this topic involves many kinematics concepts, the students' learning difficulties involving science concepts should not be overlooked. Perhaps the science and mathematics department could work together to achieve a more fruitful learning of relative velocity.

References

Abd Rahman, N. (2004). Physics Teachers Strategies and Reflective Ability for Addressing Pupils' Misconceptions in the Classrooms. *Paper presented at the British Educational Research Association Annual Conference,* University of Manchester, 16-18 Sep 2004.

Chua B.L. & Eric W. (2005). Working with Logarithms: students' misconceptions and errors. *The Mathematics Educator, 8*(2), 53-70.

Driver (1983). *The Pupil as Scientist?* Milton Keynes: Open University Press.

Gagne R. (1970). *The conditions of learning.* New Yrok: Holt, Rinehart and Winston.

Ling L.M., Chik P. & Pang M.F (2006). Patterns of variation in teaching the colour of light to Primary 3 students. *Instructional Science, 34*(1), 1-19.

Ministry of Education. (2000). *General Certificate of Education (Ordinary Level / Normal Level): Mathematics Subjects Syllabuses*, UCLES/MOE, Singapore.

Muijs, D. & Reynolds, D. (2000). School effectiveness and teacher effectiveness in mathematics: Some preliminary findings from the evaluation of the Mathematics Enhancement Programme (Primary). *School Effectiveness and School Improvement, 11*(3), 273-303.

Rosenthal R. (1976). *Experimenter effects in behavioral research*. Irvington, New York.

Toh T.L. (2003). On Using Geometer's Sketchpad to Teach Relative Velocity. *Asia-Pacific Forum on Science Learning and Teaching*, *4*(2), Article 8, http://www.ied.edu.hk/apfslt/v4_issue2/toh/

Toh T.L. (2004). *Vectors and Relative Velocity*. McGraw-Hill Education, Singapore.

Toh T.L. (2005). On In-Service Mathematics Teachers' Content Knowledge on Kinematics. *Asia-Pacific Forum on Science Learning and Teaching*, *6*(2), Article 7, http://www.ied.edu.hk/apfslt/v6_issue2/toh/

Usiskin, Z. (2001). Teachers' Mathematics: A Collection of Content Deserving to be a Field, *The Mathematics Educator, 6*(1), 81-99.

Wenglinsky, H. (2000). *How teaching matters: Bringing the classroom back into discussion about teacher quality.* Educational Testing Service: Princeton, NJ.