An instructional challenge through problem solving for physics teacher candidates

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Abstract

The teaching of science, especially at pre-college and teacher education levels has undergone tremendous transformation over the years: from teacher-centred transmission to student-centred approaches rooted in constructivism. Whereas constructivism has been charged with all manner of shortfalls, it still can be of benefit to the way physics instructions are organized and implemented. In this paper, the importance of learners' prior knowledge in understanding physics concepts is discussed. This study comprised a case of two cohorts of physics teacher candidates who had strong content knowledge of physics, but lacked pedagogical knowledge as demonstrated by their struggles to implement appropriate grade-level strategies in physics problem solving tasks (which are amenable to a variety of mathematical tool-choices). The case cohorts we used as exemplars to underscore the importance of learners' prior mathematical knowledge. Further, we focus on implications for pre-service teacher preparation, and the effects mathematical tool-choice can bear on students' conceptions.

Keywords: constructivism, physics teacher, problem solving

Background

This paper reports on the analysis of a problem solving discourse involving two cohorts of pre-service physics teachers in a University that recruits people with a minimum of a bachelor's degree in physics or related degree, such as engineering, for a one-year Bachelor of Education degree program. Lately, in the Canadian context, there has been an increase in the number of pre-service physics teachers with bachelor's and even master's level degrees in physics or engineering entering the teacher education program. This is regarded as a good thing since these are some of the previously top science students who had joined other professions because of their excellence in physics. In other words, teacher education programs in Canada are now preparing some of the top academic performers to become teachers.

This background has made it challenging for teacher education programs to critically examine the one-year teacher preparation model, as there is a widely held view that content knowledge equates to pedagogical knowledge. What defenders of this position may not appreciate is that teaching is a discipline that has rules grounded in research and scholarship. Those who join teacher education programs expect to be prepared to teach effectively. Effective teaching in part involves the ability to develop grade-level-appropriate instructional strategies. These strategies vary from one discipline to another. Physics embodies a problem-solving character that uses mathematics knowledge as a tool. But caution needs to be taken at the teacher preparation level to help pre-service teachers link their own content knowledge with pedagogical strategies appropriate for their students. Our pre-service physics teachers are highly qualified individuals who have more than enough content knowledge to teach high school students. However, subject content knowledge does not equate to pedagogical content knowledge (Shulman, 1987) or a teacher's ability to develop effective instructional strategies.

Planning for instruction, including selection of resource materials and instructional strategies is very important for effective teaching. However, one-year teacher preparation programs are short in duration, and hence, run the risk of mechanistic apprenticeship of pre-service teachers. At the University of British Columbia, our teacher candidates spend a short time (38.5 hours) on physics methods out of the 12 months spent in the Teacher Education Program. But to investigate the teacher candidates' ability to develop grade-level-appropriate problem solving strategies, at the end of each physics methods course, we have given our pre-service physics teachers a physics problem to solve as homework. We require them to use as many methods as possible. Our teacher candidates are specifically challenged to generate at least one solution procedure that employs methods the majority of high school (grade 11 and 12) students can easily understand or in which they have some knowledge or experience.

In this paper, we use one particular problem as an exemplar because of the various approaches it attracted from two cohorts of 11 and 16 pre-service physics teachers in two consecutive academic years, Y1 and Y2, respectively. Though limited to the two cohorts and this one problem, the investigation is based on our belief that through participation in a classroom discourse, pre-service teachers are provided opportunity for reciprocal learning. Moreover, engaging in this activity assisted teacher candidates' learning through socialization, allowed modeling of pedagogy, and thus elicited the teacher candidates' content knowledge including their mental stock of problem solving methods. Data for the study comes from detailed record keeping of the discussions and the exercises during the two cohorts' physics methods classes.

Group discussion discourse was employed to model what we considered to be an appropriate strategy for instructing mixed ability or diverse classes as well as offering opportunity for the teacher candidates' socialization, which is an important step to instilling into the candidates a sense and appreciation of the power of group learning. As part of our pedagogical approach to this physics methods course, we hoped that group discussion of the various problem solving approaches that were generated amongst our pre-service teachers could be provocative for their thinking about methods that they could use with their future students. Through our pedagogical method, we hope that the majority of students, and not just the few high achievers (gifted), will be “taught". It is not uncommon, for example, to find a particular problem, such as the one used in this investigation, attracting different approaches to its solution from among subject-content-rich persons (e.g., pre-service teachers). The question this raises is whether the strategies employed are appropriate to the level of the students. In particular, given that physics problem solving employs mathematics, the question is whether the mathematics employed to develop solutions is within the grasp of the students, a majority of whom do not have broadly-based background knowledge or are not “gifted". It is now widely acknowledged that building on students' prior knowledge to develop solutions to physics problems can be very fruitful in terms of student learning, thus directing us to attend to how our pre-service physics teacher candidates are socialized to consider and utilize their students' prior knowledge.

Teacher Learning Through Socialization

According to Zeichner and Gore (1990), teacher socialization is the process whereby an individual becomes a participating member of the society of teachers. As applied to this paper, teacher candidate socialization involves being a participating member of a learning group. What this means is that in belonging to the group one participates in all activities of the group including learning from other group members and through a process of reciprocal teaching (Palincsar & Brown, 1984). What is learned in such a setting can profoundly influence any future actions in activities similar to what was experienced in the group.

In other words, socialization in this paper is seen as a construction process whereby attitudes, beliefs and ways of doing things are influenced during group discussion, hence our view that we were enhancing group learning through a socialization process. An earlier study investigating teacher candidates' perceptions of the status of Physics 12 revealed that such a construction process occurred when they were high school students (Nashon & Nielsen, 2007). Teacher candidates' perspectives were in large part shaped by the experiences they had while in high school or during their undergraduate programs. Right from their experience as elementary, high school or university students the teacher candidates were immersed in the culture of teaching and learning (Lortie, 1975).

Brousseau, Book and Byers (1989) see the effects of a "teaching culture" in shaping a teacher's education beliefs as spanning school contexts. In their study of teachers, Brousseau et al. conclude that the number of years that a teacher has worked in this capacity significantly affected or influenced their beliefs on teaching. At the beginning of a careert then, it is important to help teacher candidates gain an awareness of their own beliefs about teachers alongside development of appropriate pedagogical models for use in teaching. In our view, effective pedagogy in teacher education also involves modeling learning through socialization mechanisms such as working in groups.

We do not hesitate to add that the teaching experience modeled for the teacher candidates is likely to have profound influence on their future instructional practice. Teaching culture is conveyed and experienced differently by teacher candidates as they attend classrooms with so many instructors, and this varies to some extent depending on the subject area (Burden, 1990). In the same vein, Behar-Horestein, Pajares and George (1996) consider teaching beliefs as affecting students' learning behaviour. With this background, it can be argued that instructional models used in teacher education programs are likely to influence the teacher candidates' teaching behaviour when they become teachers, and this behaviour is further influenced by their own prior knowledge.

The Role of Prior Knowledge

Development of grade-level-appropriate problem solving strategies or approaches as much as is feasibly possible should, among other things, draw from students' prior knowledge. As Kelly (1955) has said: "All thinking is based, in part, on prior convictions. A complete philosophical or scientific system attempts to make all... [this] prior knowledge explicit" (p.6). What Kelly seems to be suggesting is that prior knowledge plays a prominent role in human attempts to interpret experience. Further, understanding new concepts involves the reconstruction of incoming information in terms of prior knowledge held by the individual, and prior knowledge can be replaced or reconstructed. In the same vein, Novak and Gowin (1984) echo this view by describing the distinction between meaningful and rote learning:

To learn meaningfully, individuals must choose to relate new knowledge to relevant concepts and propositions they already know. In rote learning... new knowledge may be acquired simply by verbatim memorization and arbitrarily incorporated into a person's knowledge structure without interacting with what is already there (p. 7).

Consistent with the above citation, Bodner (1986) quotes Ausubel on prior knowledge: "If I had to reduce all of educational psychology to just one principle I would say this: The most important single factor influencing learning is what the learner already knows" (Bodner, p. 877).

We see a trend of scholars and researchers all trying to underscore the role that students' prior knowledge plays in learning. Already this paper underscores this condition of learning. Ausubel (1968) delineates meaningful learning from the rest saying: "It is apparent... that insofar as meaningful learning outcomes in the classroom are concerned, the learners' cognitive structures constitute the most crucial and variable determinants of potential meaningfulness" (p. 40). This way of acquiring knowledge is what has come to be known as constructivism. Theories that guide this method of learning subscribe to the view that knowledge is constructed and not just merely added. Hodson (1998) sheds more light on what constructivist theories of learning are all about. Learning is about the process of eliciting, clarifying and constructing new ideas, all of which take place in the mind of the learner. It seems therefore in the interest of good pedagogy to elicit students' prior knowledge, which very likely will influence the understanding or meanings accorded to new concepts or experiences (Ausubel, 1963, 1968; Bodner, 1986, Kelly, 1955; Novak & Gowin, 1984).

The power of prior knowledge in influencing conceptual understanding or practice has been underscored in contemporary research and literature in science education. We have even seen this evidently reflected in successful analogies - those that employ the use of ‘knowns' to explain ‘unknowns' (Nashon, 2000, 2001, 2004a, 2004b). The process of building on prior knowledge involves a reconstruction of the already possessed knowledge systems (accommodation), where the existing knowledge is inadequate to explain new encounters/phenomena or filling gaps (assimilation) within the existing knowledge systems (Posner, Strike, Hewson, & Gertzog, 1982). This creates a cognitive conflict where the learner is challenged to fit the incoming information into the existing knowledge (Gunstone, 1992). But the question remains whether all students' prior knowledge is “acceptable" to the scientific cannons of physics. Some ‘knowledge' could be counter-physics, or even, “un-physics."

It seems to make good pedagogical sense for the physics instructor to identify counter physics ideas and target them by providing experiences in which such “un-physics" ideas get challenged. In fact, according to Hodson (1998), “Secure conceptual understanding is the ‘trigger" for changing the language and for making progress towards more sophisticated understanding" (p. 24). Thus, in the current paper, apart from illuminating the challenges pre-service teachers confront in developing grade-level-appropriate problem solving strategies, which is a skill we believe must be deliberately taught and exemplified, we aimed to reiterate the importance of prior knowledge in learning. And given the fact that mathematics is a tool of physics (Von Weizsacker & Juilfs, 1957), there is strong evidence to suggest that physics instructors should utilize students' relevant prior knowledge in explaining solutions to intended physics problem solving tasks. This requires a great deal of preparation and thinking, which is consistent with developing teachers' pedagogical content knowledge [PCK] through their professional training and practice.

Through the process of becoming qualified teachers, the problem solving strategies modeled in this study would then be part of the teachers' PCK (Shulman, 1986), including their understandings of the connections between physics and mathematics. In a recently concluded study about the status of Physics 12 in British Columbia, the physics teacher and teaching styles were prominently mentioned as impacting students' decisions about Physics 12 (Nashon, 2005). In the same vein Blanton (2003) and Kumagai (1998) indicated that quite often science teachers conform to instructional models they were exposed to as high school students. In retrospect, this, in part, constitutes the kind of pedagogical knowledge that is constructed or modeled in the context of teacher candidates' problem solving tasks that could shape their view of the nature of problem solving in physics.

Connection Between Mathematics and Physics

According to Weizsacker and Juilfs (1957), “The tool of conceptual thought in physics is mathematics, for physics treats the relations measured, which is numerically determined, magnitudes" (p. 11). The connection between mathematics and science (physics included), is further expressed by Kline (1980): “Science must seek mathematical description rather than physical explanation. Moreover, the basic principles must be derived from experiments and induction experiments" (p.51). This is the principle on which Newton and Galileo operated, and in which contemporary thought in physics still resides. The importance of mathematics in physics classes is evident and is perceived so by instructors of physics. Curriculum materials portray a similar image: it is virtually inconceivable to have a page in a physics textbook ending without a single equation or other form of mathematical expression.

In some cases physics is synonymous with mathematics. In short, mathematics is important as far as physics is concerned, but with due respect to other forms of knowledge domains. But, as far as physics is concerned, mathematics constitutes a large portion of its language. What is troubling though, is the fact that some instructors of physics seem to recognize this importance and yet never make deliberate effort to sharpen their physics students' mathematical knowledge needed for the moment - a moment when the mathematical knowledge appropriate to the teaching of the intended physics concept is required.

Arguments exist about whether or not physics can be taught without the use of mathematics (e.g. Tao, 2001). Nonetheless, it is almost impossible to imagine complete physics knowledge without its quantitative aspects (Nashon, 2006). In other words, it appears almost a given fact that the physics knowledge domain is constructed through both qualitative (involving observation and description) and quantitative (involving measurements and calculations) methods.

The current investigation centred on quantitative methods of problem solving, since these employ mathematics as a tool for use in the process of physics knowledge construction. The challenge is how to teach high school students, many of whom may not be as proficient at using this tool of physics as we would want to assume (Basson, 2002; Nashon, 2005, 2006).

Many problem solving tasks in physics are characterized by the use of equations and other forms of formulae. In our view, students coming to physics classes where instructions utilize knowledge of equations they already know experience minimum obscurity of the intended physics concepts by the mathematics. Conversely, if too much new information is to be learned concurrently or over too short a period of time, students may experience cognitive overload. Of course, we might partly appreciate this, in a metacognitive sense (Gunstone, 1992; Nashon & Anderson, 2004), as we consider fundamental issues such as those underscored by Sherin (2001):

What does it mean to understand an equation? The use of formal expressions in physics is not first, a matter of rigorous and routinised applications of principles, followed by the formal manipulations to obtain an answer. Rather, successful students learn to understand what equations say in a fundamental sense; they have a feel for expressions, and this guides their work (p. 479).

Because the majority of high school students lack a depth of mathematical competency, understanding of physics tends to be obscured by the students' attempt to understand the mathematics, which is used to develop the logical arguments that bring about understanding of the intended physics concept(s), which Ausubel (1963, 1968) describes as meaningful leaning. The lack of mathematical competency could lead to an over-emphasis on qualitative methods by physics teachers. But, such an approach would necessarily be limited in scope, since certain aspects of physics explanations are rooted in and explicated through the mathematics knowledge domain (Nashon, 2006). It is for this reason that the current paper reports on an investigation aimed at determining pre-service teachers' ability to appropriately employ problem solving methods that utilize mathematical tools that are suitable to the grade level of students they are teaching.

There is a question as to whether physics instructors discern what prior mathematics knowledge their physics students possess so as to apply it to the intended physics concept. This paper underscores the idea of sensitizing pre-service physics teachers during their initial teacher preparation as to the need to always build on students' prior knowledge. Therefore, as an attempt to underscore the importance of this aspect of pedagogy, in the current investigation, we used a problem solving task that drew from the kinds of knowledge that the majority of high school students already possess, and which is open to a variety of problem solving approaches. Through this pedagogical approach, we examined our pre-service teachers' ability to use mathematically modeled strategies that were appropriate and relevant to grade 11 and 12 physics teaching. The intention was to sensitize the pre-service physics teachers to the fact that, although all possible approaches could lead to the same answer, some high school students may not understand all of the approaches. Hence the questions for the current study: What problem solving strategies does a physics problem challenge elicit from teacher candidates with content-rich physics backgrounds? Which of the strategies are appropriate to grade 11/12 levels and employ grade-level-appropriate mathematics? What pedagogical implications does this experience offer?

The Study

The study examined a case of two cohorts of pre-service physics teachers from two consecutive academic years in the teacher education program at the University of British Columbia. Each cohort was assigned the same particular problem solving task, that we saw as amenable to a variety of problem solving approaches, with the expectation that the teacher candidates would generate as many approaches as possible, including at least one that drew from mathematics knowledge familiar to the majority of grade 11 and 12 students.

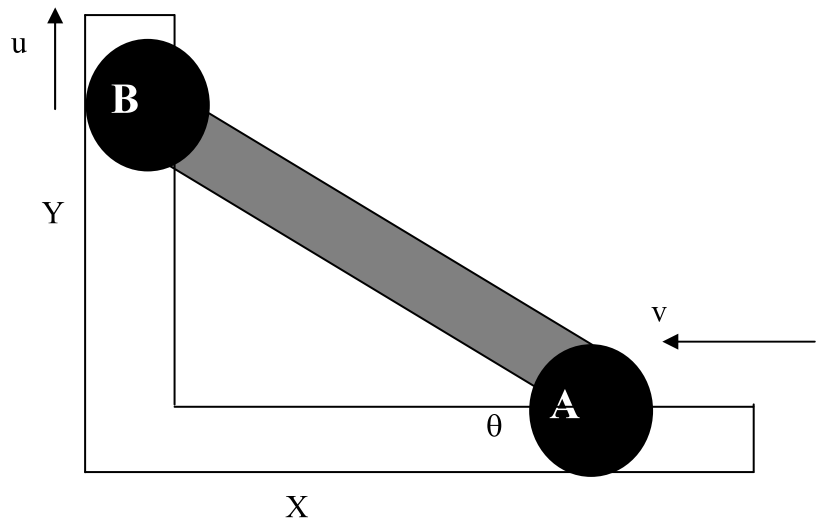
Firstly, the pre-service physics teachers solved the problems individually. Then, they were asked to note similarities and differences as they shared their solutions in groups of three or four. Each group compiled successful problem solving approaches as developed by individual members, which were then presented on overhead transparency sheets to the rest of the class for discussion. The challenge our teacher candidates faced was to generate several problem solving approaches, at least one of which would utilize mathematics knowledge that would be familiar to the majority of grade 11 and 12 high school students. In other words, the problem solving approaches should be derived, as much as possible, from the mathematics knowledge and skills possessed by or familiar to students in grades 11 and 12.

We chose the problem for this assignment based on how challenging and amenable it was to different problem solving approaches. We saw possible solution pathways that ranged from the use of the more advanced to less advanced mathematics knowledge and skills, which in turn was used as a way to evoke and elicit a variety of methodological schema among our pre-service physics teachers. Thus, we used a problem of a slightly more advanced level than is normally found in high school physics in order to provide enough challenge for our teacher candidates, and at the same time generate the level of discussion that illustrated our intended message: teachers need to be sensitive to students' prior knowledge levels. Our teacher candidates recorded their problem solving approaches on transparencies, which along with discussion transcripts, were subsequently used for our in-depth analysis. In this paper, we categorized the solution approaches according to a schema that represented the physics and mathematical knowledge found in each problem solving approach.

The problem

A rigid rod that has length L connects two objects, A and B. The objects slide along perpendicular guide rails as shown in Figure 1 below.

Figure 1. *The physics problem*



Adopted from:
Physics for Scientists and Engineers, 5th Ed
by Serway, R. A & Belchner, R. J. (2000, p.56)

Results

There were 11 physics teacher candidates from Year One (Y_1) and 16 from Year Two (Y_2) who participated in the study. A total of nine different problem solving strategies or approaches were discerned from the problem solutions that individuals had shared within their groups and then presented to class - with each of the methods leading to the same correct answer (Solution Approaches are detailed in Table 1). Through our analysis, these solutions revealed the advanced nature of mathematical content knowledge possessed by our teacher candidates. Although the data is insufficient to make a claim about their physics content knowledge, the fact that all of them succeeded at solving the problem is an indication of their capabilities in both physics and mathematics content knowledge. Given that the task was aimed at eliciting the teacher candidates' pedagogical content knowledge, the data indicate that most of them used problem solving strategies that were beyond grade 11 and 12 levels. The sharing of individual solutions in groups enabled recognition of similarities and differences in the generated problem solving approaches and offered an opportunity for collective knowledge building. Further, this served our purpose within this physics methods course, as it was consistent with our modeling of pedagogy appropriate for physics problem solving with our physics pre-service teachers.

An important aspect of pedagogical content knowledge is the teacher's ability to become aware of and elicit students' prior knowledge, which is a key influence on an individual's understanding of new experiences. In the same vein, the teacher candidates' prior knowledge was evoked by the problem solving task in the current study. The various approaches that were presented indicated prior knowledge that these highly qualified individuals brought to the problem solving task in their physics methods course. Further, through their own analysis of the problem solving approaches used by their colleagues, a discussion was engendered about the approaches and their suitability or unsuitability with the intended grade levels of physics students.

All of our teacher candidates arrived at the correct answer using their various problem solving approaches. It is notable that only two of the approaches generated by our teacher candidates were at an appropriate grade-level for high school students. Both of the approaches were non-calculus (NC) solutions and were generated by teacher candidates from the Y_1 group: there were no NC solutions generated from the Y_2 cohort. A total of seven problem solving approaches used calculus (C). This seems to confirm the view that our teacher candidates relied primarily on their own physics knowledge and did not attend to that of their high school students. It was their own conceptions of physics knowledge that the teacher candidates brought to the problem solving task that influenced their perception of the teaching task. While the non-calculus approaches utilized mathematics knowledge that can reasonably be considered familiar to the majority of grade 11 and 12 students and would most likely have been covered in previous mathematics classes. This points rather strongly to the need to consider grade-level and developmentally appropriate strategies in instructional planning, in a way, "thinking down" to a high school level.

As shown in the frequency distribution table (see Table 2), Approach C_3 occurred with the highest frequency. Approach C_2 started with similar facts to C_3 and had the second highest frequency. The differences in all seven calculus strategies/approaches lie in the starting (basic) facts and more so in the subsequent variable reorganization and manipulations. Most of the strategies (both Cs and NCs) begin from the basic facts of trigonometry for resolving the motion of the rod into its vertical and horizontal components, a mathematical skill that is familiar to most grade 11 and 12 physics students. But NC_1 and NC_2, though challenging in terms of mental "visualization," include ideas or facts from prior mathematics experience, and it can be inferred that the mathematics and physics concepts are interwoven for the pre-service teachers who used these approaches, a sophistication of thinking that is relatively uncommon among high school students. The rest of the reasoning is actually qualitative physics. For example, realizing that the rod is not moving in the direction along its length, and realizing that a component of u, ucosθ, is equal to a component of v, vsinθ constitutes prior physics knowledge. Apparently, approaches NC_1 and NC_2 are different only in their sketched diagrams (see Table 1), but the concept seems to be the same, since they both utilize the trigonometric relationship that equates ucosθ and vsinθ.

Approach NC1 appears to have less "noise" than NC_2, which in this paper means that irrelevant information or steps are included in NC_2 that have a potential to interfere with a student's understanding of the concept(s) being verified or explained/illustrated (Johnstone & Wham, 1982). However, the noise in NC_2 reveals more useful physics than what is conveyed in the calculus strategies (C_1 through C_7). The amount of detail in NC_2 could be considered useful physics, arguably adding to the students' physics content knowledge

An interesting observation was made among the majority of our participating teacher candidates. Most of them attempted unsuccessfully to use "the sum of the square of the sides of a right angled triangle" as a starting fact in their solution approaches. Many got into difficulties when attempting to factor out L and introduce u and v into the equations. Only one member of the Y_1 cohort and five members of the Y_2 cohort succeeded in utilizing this idea (labeled solution approach C_4) by injecting the idea of limits and more calculus. C_4 is an approach that could be considered to contain more 'mathematics noise' than other solutions. In other words, it contained unnecessary mathematical detail that could easily obscure understanding of the intended physics concept. Of course there is nothing wrong with utilizing this approach as long as the students possess the relevant mathematical background knowledge to understand the physics content.

The problem used in this study drew particular attention to mathematical knowledge that could be useful in solving the problem. Further, it enabled our teacher candidates to consider the nature of messages conveyed in the physics being taught in high school classrooms. Any knowledge of mathematics demanded or utilized by the teachers in physics problem solving tasks determines the physics understanding students can gain from such tasks. In other words, it is not the calculus that we are calling into question, but rather, the mismatch between physics problem solving approaches and the students' prior mathematical skills and knowledge as practised or experienced in earlier grade-level courses. The gap between teachers' understanding and students' prior knowledge is an important intersection for student learning. Using mathematical knowledge that students already have most probably will minimize cognitive noise (such as was seen in solution approaches such as NC_2 and C_4) that could obscure the physics messages intended through the problem solving activities. Coming full circle, this is consistent with the principle of attending to students' prior knowledge as a basis for meaningful learning (Ausubel, 1963).

Table 1. *Solution Approaches*

C_1

$$\mathrm{Sin}\,\theta = \frac{Y}{L};\ \mathrm{Cos}\,\theta = \frac{X}{L}$$

$$v = \frac{dx}{dt} = L\frac{d}{dt}(\cos\theta) = -L\sin\theta.\frac{d\theta}{dt}$$

$$u = \frac{dy}{dt} = L\frac{d}{dt}(\sin\theta) = L\cos\theta.\frac{d\theta}{dt}$$

$$\frac{v}{u} = \frac{-\sin\theta.\frac{d\theta}{dt}}{\cos\theta.\frac{d\theta}{dt}}$$

$$v\cos\theta = -u\sin\theta$$

$$\mathbf{u = \sin\theta.\frac{-v\cos\theta}{\sin\theta}}$$

$$\mathbf{u = -0.577v}$$

C_2

$$x = L\cos\theta$$

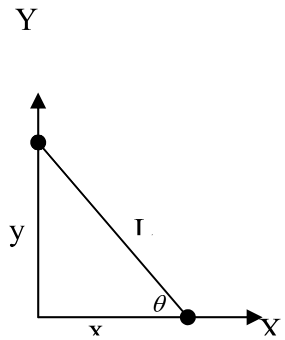
$$y = L\sin\theta$$

$$u = \frac{dy}{dt} = \frac{dy}{d\theta}.\frac{d\theta}{dx}.\frac{dx}{dt}$$

$$= L\cos\theta.\frac{1}{-L\sin\theta}.\frac{dx}{dt} = \frac{\mathrm{Cos}\,\theta}{-\mathrm{Sin}\,\theta}.\frac{dx}{dt}$$

$$\mathbf{u = -\cot\theta.\frac{dx}{dt} = -(\cot\theta)v = -0.577v}$$

C_3



$$y = L\sin\theta;\ x = L\cos\theta$$

$$v = \frac{dx}{dt} = L\frac{d}{dt}(\cos\theta) = -L\sin\theta.\frac{d\theta}{dt}$$

$$u = \frac{dy}{dt} = L\frac{d}{dt}(\sin\theta) = L\cos\theta.\frac{d\theta}{dt}$$

$$\text{Hence } \frac{d\theta}{dt} = \frac{-v}{L\sin\theta};\ \frac{d\theta}{dt} = \frac{u}{L\cos\theta}$$

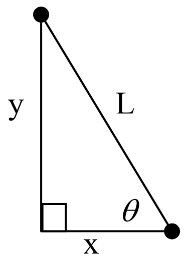
$$\text{Thus, } \frac{u}{L\cos\theta} = \frac{-v}{L\sin\theta}$$

$$u = \frac{-v}{L\sin\theta}.L\cos\theta$$

$$= -v\frac{\cos\theta}{\sin\theta} = -v\cot\theta$$

$$u = -0.577v$$

C_4



$(i)\ \frac{y}{x} = \tan\theta$

$(ii)\ y^2 + x^2 = L^2$

$(iii)\ (y + \Delta y)^2 + (x - \Delta x)^2 = L^2$

(iv) Subtracting (ii) from (iii): $2y\Delta y + \Delta y^2 - 2x\Delta x + \Delta x^2 = 0$

(v) Divide (iv) by $\Delta y : 2y + \Delta y - 2x\frac{\Delta x}{\Delta y} + \Delta x\frac{\Delta x}{\Delta y} = 0$

(vi) Therefore, $2y + \Delta y = (2x - \Delta x)\frac{\Delta x}{\Delta y}$

(vii) But, $\frac{\Delta x}{\Delta t} = v, \frac{\Delta y}{\Delta t} = u$, hence, $\Delta x = v\Delta t, \Delta y = u\Delta t$ and substituting in (

(viii) $2y + u\Delta t = (2x - v\Delta t)\frac{v\Delta t}{u\Delta t}$, *simplifying*:

$(ix)\ 2yu + u^2\Delta t = 2xv - v^2\Delta t$

(x0 Limits $\Delta t \to 0, 2yu = 2xv$

(xi) Therefore, $u = \frac{2x}{2y}.v = \frac{x}{y}.v = \frac{1}{\tan\theta}.v = (\cot\theta)v = (\cot 60^0)v$

(xii) $u = 0.577v$.

C_5

$$\frac{y}{x} = \tan\theta,\ y = x\tan\theta$$

$$x = L\cos\theta,\ y = L\sin\theta$$

$$\frac{dy}{dt} = \frac{d}{dt}(x\tan\theta) = \tan\theta.\frac{d\theta}{dt} + x\frac{d}{dt}(\tan\theta)$$

$$= \tan\theta.\frac{dx}{dt} + x.\sec^2\frac{d\theta}{dt}$$

But $\frac{d\theta}{dt} = \frac{dx}{dt}.\frac{d\theta}{dx},\ \frac{dx}{dt} = v$

Therefore $\frac{d\theta}{dt} = v\frac{d\theta}{dx}$, But $x = L\cos\theta$ and $\frac{dx}{d\theta} = -L\sin\theta$

Thus $\frac{d\theta}{dt} = v.\frac{1}{-L\sin\theta} = \frac{-v}{L\sin\theta}$

But $\frac{dy}{dt} = u = \frac{d}{dt}(x\tan\theta) = x\frac{d(\tan\theta)}{dt} + \tan\theta\frac{dx}{dt}$

$$= v\tan\theta + x.\sec^2\frac{-v}{L\sin\theta} = v\tan\theta - \frac{vx.\sec^2\theta}{L\sin e\theta}$$

$$= v\tan\theta - \frac{v.\sec^2\theta}{\tan\theta} = v\tan\theta - \frac{v(1+\tan^2\theta)}{\tan\theta}$$

$$\mathbf{u = v\tan\theta - v\cot\theta - v\tan\theta}$$

$$\mathbf{u = -v\cot\theta = -v(\cot 60.0^0)}$$

Therefore, $\mathbf{u = -0.577}v$

C_6

$$x = L\cos\theta,\ v = \frac{dx}{dt} = L\frac{d}{dt}(\cos\theta)$$

$$y = L\sin\theta,\ u = \frac{dy}{dt} = L\frac{d}{dt}(\sin\theta)$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx}$$

But $u = \frac{dy}{dt} = \frac{dy}{d\theta}.\frac{d\theta}{dx}.\frac{dx}{dt}$

$$= \frac{dy}{d\theta}.\frac{d\theta}{dx}.v$$

$$= \frac{L\cos\theta}{-L\sin\theta}.v$$

$$= \frac{\mathbf{\cos\theta}}{\mathbf{-\sin\theta}}\mathbf{.v = (-\cot\theta)v}$$

Therefore, **u = 0.577v**

C_7

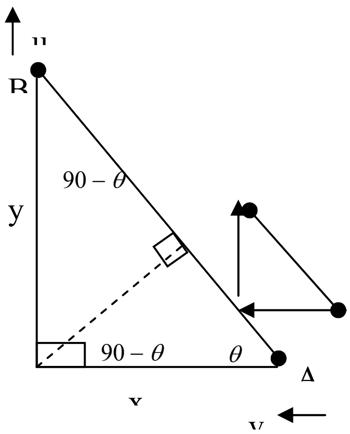
$$x = L\cos\theta,\ v = \frac{dx}{dt} = -L\sin\theta).\frac{d\theta}{dt}$$

$$y = L\sin\theta,\ u = \frac{dy}{dt} = L\cos\theta.\frac{d\theta}{dt}$$

$$\frac{u}{v} = \frac{L\cos\theta.\frac{d\theta}{dt}}{-L\sin\theta).\frac{d\theta}{dt}} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

$\therefore \mathbf{u = -0.577v}$

NC₁



Since there is no movement in either AB or BA

direction of the rod,

$u\cos(90-\theta) + v\cos\theta = 0$

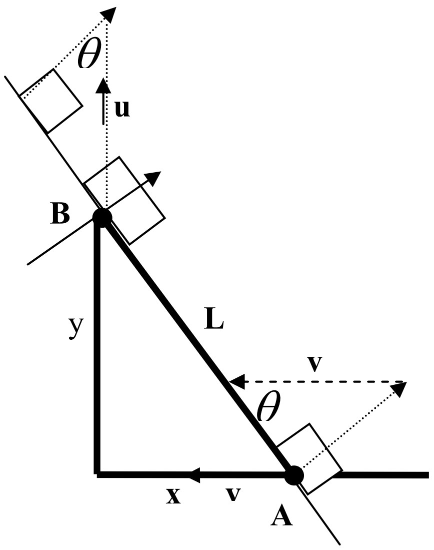
$$u\sin\theta = -\,v\cos\theta \Rightarrow u = \frac{-\cos\theta}{\sin\theta}.v$$

$$\text{but } \theta = 60^0$$

$$u = \frac{-\cos 60^0}{\sin 60^0}.v = -\,0.577v$$

$$u = -\,0.577v.$$

NC₂



Because the resultant velocity in either AB or BA direction of the rod is = 0,

$$u\sin\theta + v\cos\theta = 0$$

$$u\sin\theta = -v\cos\theta$$

$$u = \frac{-\cos\theta}{\sin\theta}.\mathbf{v} = -\,v\cot\theta = \mathbf{-0.577v}$$

Table 2. *Frequency Distribution of Solution Approaches, by Year*

Approach	C_1	C_2	C_3	C_4	C_5	C_6	C_7	NC_1	NC_2
Y1	2	4	3	1	3	0	3	1	1
Y2	4	2	5	5	0	3	0	0	0
Total	6	6	8	6	3	3	3	1	1

Discussion

Highlighting key points of the group discussion, our analysis raises five issues: 1) the problem attracted nine strategies/approaches (two non- calculus, seven calculus); 2) non-calculus approaches utilized math that could be considered familiar to grades 11 and 12 students; 3) consistent with a constructivist view, our teacher candidates' prior knowledge appeared to have strongly influenced their conformity or lack of conformity with the nature of desired outcome of the task; 4) our teacher candidates' solutions seem to convey the impression that the math and physics concepts imbedded in the problem are interwoven; and, 5) the relevance or irrelevance of mathematical noise in a solution depends on the extent of obscurity or clarity of the intended physics concepts.

Further, it is important that groups be trained to be critical of their own learning or practice. Also, on the one hand, majority of the groups (with calculus solutions) did not reflect on their solutions as they only thought in terms of calculus. On the other hand as a class they learned from different group presentations, including solutions that conformed to the expectation of the problem solving task. The group discussions were the place where this sort of learning could happen.

Problem Solving Strategies and Prior Knowledge

The study's results, though not necessarily applicable to every pre-service physics teacher or to high school physics teachers in general, nonetheless highlight the need to make the match between grade-levels and problem solving strategies a key part of our pre-service physics methods curriculum, and further, the need to underscore the role students' prior mathematics knowledge plays in their understanding of the knowledge intended to be conveyed through physics problem solving tasks. We modeled this pedagogical approach with the assigned problem solving task and provided opportunity for prior pedagogical content knowledge to be elaborated within the group discussions.

Although questions could be raised about whether or not the assigned problem was suitable for a grade 11 or 12 classroom, it nonetheless served the purpose of the investigation, which was to determine the teacher candidates' ability to generate problem solving strategies appropriate to grades 11 and 12. That is, we deliberately chose this problem because of the challenge it offered to our teacher candidates. We wanted to influence our teacher candidates to always endeavour to build upon what their students already know. It was our view and experience that using a grade 11/12 level problem did not provoke the kind of discussion and challenge that was needed to reflect real life experiences. More particularly, content-rich individuals (e.g. physics teachers) quite often tend to underestimate the difficulty their students experience when what is being taught does not relate to what is already known. Moreover, our goal of provoking thought, engagement and offering real challenge would have been undermined by using a problem at a grade 11/12 level of difficulty.

The problem solving task evoked the teacher candidates' prior mathematics content knowledge and the extent to which they could apply it to solve the problem. In other words, the problem or task helped elicit the teacher candidates' pedagogical content knowledge, which according to Shulman (1986) refers to, "the ways of representing and formulating the subject that makes it comprehensible to others" (p. 9). However, this paper adopts an elaborated version in which PCK includes teachers' interpretations and transformations of subject matter knowledge in the context of facilitating student learning. Simply put, Shulman (1987) considers pedagogical content knowledge as the category most likely to "distinguish the understanding of the content specialist from that of the pedagogue" (p. 4). Pedagogical content knowledge thus depends on complex interactions between discipline knowledge, pedagogic knowledge, and the teacher's experiences in teaching that knowledge (Cochran & Jones, 1998; Tobin, 1998; Tobin, Tippins & Galland, 1994). We believe that these interactions need to be explored and reflected upon during pre-service teacher education so that teacher candidates' developing pedagogy includes eliciting and building upon their students' prior knowledge.

Candidates' Pedagogical Content Knowledge (PCK)

Many of the teacher candidates could not generate problem solving approaches that conformed to the challenge of the assigned problem solving task, but the task most likely evoked an awareness of what they did or did not know. This interpretation falls within the realm of constructivist theories of learning. In other words, the understandings the candidates developed are according to Kelly (1955),

> Ways of constructing the world. They are what enables…[one] to chart a course of behavior, explicitly formulated or implicitly acted out, verbally expressed or utterly inarticulate, consistent with other courses of behavior or inconsistent with them, intellectually reasoned or vegetatively sensed (p. 9).

It should be clear that the prior knowledge physics students possess at the time new concepts are being taught (whether through problem solving tasks or some other means, such as experimentation) is a major factor in determining the ease with which they understand the new concepts and the pace at which the teacher covers the intended content. Problem solving, as we have already argued, uses mathematics in the modeling of solutions to problem solving tasks. Absent, inadequate or poorly understood prior mathematics knowledge might necessarily inhibit a student's understanding of the intended physics concepts. Use of students' prior mathematical knowledge will most likely enhance their understanding of the intended physics concepts. Teaching or activating the mathematics knowledge will then allow the students to concentrate on the physics concepts since the mathematical skills will be familiar. We see this linkage as an important objective for instructional planning: helping students to build substantive understanding across subject areas, freely utilizing different ways of knowing to deepen and broaden conceptual knowledge structures. This is the pedagogy that was being modeled through the problem solving task, and physics instructors should where possible, utilize math content with which the students are already familiar, thereby reducing cognitive overload. Also it is important to minimize unnecessary mathematics noise, even when it is familiar to the students.

Noise-Laden Strategies

If the purpose of a lesson is to teach physics concepts, it makes good pedagogical sense to minimize any impediments, such as "mathematics noise" (Johnstone & Wham, 1982), to students' understanding since there is always a high possibility that employing complicated mathematics in physics problem solving tasks will obscure understanding of the intended physics concepts. Otherwise, those who concurrently succeed in understanding both the mathematics and relevant physics are likely to be in the minority. Several problem solving strategies developed by our teacher candidates contained mathematics noise. These strategies can expose students to “cognitive overload." Although our teacher candidates were asked to generate strategies that used mathematics familiar to grades 11 and 12 students (which calculus-based strategies C_1 through C_7 are not), C_1 through C_3 and C_7 have less mathematics noise than C_4 through C_6. It is even more overloading if the noise involves unfamiliar mathematics. However, if the noise is familiar physics, then in a way such noise might serve a useful purpose in some cases, for example, remediation. In order to minimize cognitive overload or mathematical noise, it might be helpful to the students if physics teachers provided remediation on the necessary mathematics knowledge and skills that may be required for particular physics units or topics prior to launching into the teaching of the physics concepts. Furthermore, since it is widely understood and also acknowledged in this paper that mathematics is a tool of physics, we see it as imperative to impress upon physics instructors the need to utilize students' relevant prior knowledge in explaining or working out solutions to physics problems. This might require a great deal of preparation and thinking - such as what was experienced by the teacher candidates during the problem solving task in this study. In fact, it might be easier for students to understand physics concepts conveyed through mathematics that the majority of students already possess. Realization of this fact and using it to develop instructional strategies that build upon appropriate mathematics content requires deliberate attention and modeling at the teacher preparation level. A further aspect of our teacher candidates' preparation involved their learning through group socialization.

Teacher Learning and Socialization

The kinds of experiences that our teacher candidates encountered during the physics problem solving activities are what this paper considers to be consequences of learning group socialization. Although individual teacher candidates initially generated the solution strategies in this study, they in turn shared them in groups and presented to class what they had agreed upon to be strategies represented within the group. In the process of sharing their strategies, there were knowledge exchanges between group members. In a way this was modeling teacher-learning communities (Lave & Wenger, 1991).

In a similar way, it is hoped that by engaging and wrestling with tasks that challenged them to generate strategies that are appropriate to grades 11 and 12, the teacher candidates experienced and constructed a feel for the student learning experience. The data in Table 1 are about individual as well as group products (problem solving strategies). Moreover, these data were generated within a group learning context, which in many respects involved sharing ideas, having a common experience and appreciating the challenges involved in planning appropriate grade-level strategies. This made our teacher candidates part a socialization process, hence the idea of teacher candidate socialization.

As reported in this study, problem solving strategies developed by individuals were shared in groups and presented to the whole class as group presentations. We believe our task as teacher educators is to help our teacher candidates build and rebuild what they already know about the work of teachers (Feiman-Nemser & Floden, 1986), and this is only possible for preservice teachers when they are put in situations where pedagogy is modeled. We also believe that such modeling is more effective if it involves a socialization process such as working in groups where they share information, experience, challenges and thoughts.

Modeling pedagogy

In this paper, it is argued that teacher candidates construct their images or perceptions of teaching in a particular discipline through direct or indirect socialization. The use of the term direct here is intended to mean a deliberate effort made by teacher educators to have the pre-service teachers adopt, practise and accept certain ways of experiencing a subject. Correspondingly, indirect refers to an unknowing on the part of the teacher educator, whereby the teacher candidates, by experiencing the teacher educator's way of doing things, can come to believe that that is the way things are done or work in that subject. We could further hope that our teacher candidates have thus reconstructed their own images of teaching and that they will teach how they were taught to teach (Blanton, 2003; Goodlad, 1984; Nashon, 2005). The problem solving task in the current study was intended to do just that: model pedagogy where the would-be teachers will elicit their own students' prior knowledge of what they intend to teach using strategies similar to what was used in this study including the use of a problem to evoke as well as challenge existing knowledge. Further, through capturing this knowledge in records such as written assignments, individuals have a frame from which to share their understanding in a group context or social setting. The particular problem used as a context for this type of assignment must also be amenable to multiple interpretations or approaches in order to generate constructive argumentation, for example, such that each person has opportunity to explain their problem solving approach. Hansen (1995) argued that this was specifically the case with technology education students who had come from a variety of business and industrial backgrounds to their teacher education programs, through which were conveyed differing notions of what it is to be a technology teacher, models that may have been inconsistent with those in use in the teacher education program. The teacher education program then, needed to challenge these various perspectives in order that the candidates could be socialized into the culture of teaching.

In this study there is a high possibility that the candidates were driven by the desire to solve the problem and not necessarily to teach the problem. Thus, we argue that the problem was not just aimed at challenging their desire to solve the problem, but also to orient them toward a desire to generate solution strategies appropriate for grades 11 and 12 students. In other words, they were challenged to not merely solve the problem but to engage in a pedagogical process of deciding and generating solution strategies that are appropriate for the grade levels they will teach. But we wish to acknowledge the fact that even the non-calculus strategies involved very subtle non-standard, non-mathematical solutions or reasoning that could be quite challenging for grade 11 and 12 students. Also, our teacher candidates, as we already pointed out, could be classified as experts and it is widely acknowledged that experts tend to quickly characterize problems as being of particular types (Goldman, Petrosino & Cognition and Technology Group, 1999). Thus, once the majority of teacher candidates classified the problem in this study as requiring calculus, they found it difficult to reason beyond this categorization and return to the needs and abilities of grade 11 and 12 students. Through the assigned problem, the study aimed at to raise our teacher candidates' awareness of this difficulty. The fact that the challenge raised our teacher candidates' awareness is to us very important in terms of pedagogy. Of course there are some teacher candidates who were in a way oblivious to the kind of math appropriate to grade 11 and 12 levels and how the problem could be solved without knowledge of calculus. But, it was through the group discussions and class presentations that we used as pedagogical tools in our physics methods course to draw attention to our teacher candidates' use of their own prior knowledge and the mismatch with the levels of both mathematics and physics knowledge among their high school students. We see this as an important strategy in teacher education.

The pre-service teachers' views of school physics appear to have been influenced by how they understood physics and its teaching. In addition to what a number of studies (e.g. Dweck & Bempechat, 1983; Fisher et al, 1978) have revealed about orientations to teaching influencing teacher decisions and actions, Bunting (1984) proffers that assuming a variance between teacher beliefs and teacher behaviours, knowledge of the content of beliefs becomes an important first step in the identification of variables within the educational context which mediate between the thinking and practice of teachers.

Conclusion

The majority of teacher candidates used very advanced calculus to generate a solution to the assigned problem for the study reported here. These solutions were certainly not within the realm of grade 11 and 12 students' grasp, even though our teacher candidates were asked to consider grade-appropriate solution paths. Many of the strategies embodied mathematics noise (Johnstone & Wham, 1982) and further, most of the strategies could lead to cognitive overload for the majority of grade 11 and 12 students. The findings discussed in this paper point to a need to pay attention to pedagogy as well as content necessary to instruct high school physics. The framing of this investigation resulted in consequences for modeling group learning and pedagogy, and further, the need to explicate teacher candidates' pedagogical content knowledge. This later point should be the apex of the physics methods courses in teacher education programs.

Recommendations

Part of a student’s prior knowledge in physics problem solving tasks should be mathematical knowledge and skills relevant to the intended physics content. In case the students lack the requisite mathematics competencies, physics teachers should feel obliged to facilitate the “sharpening" of these skills by providing remediation. It is incumbent upon physics teachers to employ problem solving approaches that utilize mathematical tools that are appropriate to their students' grade-levels and not be driven by the desire to merely solve the problem irrespective of students' prior understanding of the mathematics employed. These rote approaches are unsatisfactory for the more conceptually challenging types of problem solving that we want our students to be able to do. If any mathematics competencies are necessary for physics problem solving tasks, it would be prudent to teach these competencies first and then, through much simpler examples that could provide a bridge to the main problem solving task, introduce the students to the physics problem. Such mathematics competencies should not be too complex since the mathematics could obscure the message intended through the physics problem solving task.

Where such mathematics competencies prove to be complex, enough time should be given for students to practise the mathematics skills until some level of proficiency is noted. In cases such as this, we recommend that students be given practice exercises that largely involve real life situations for the purpose of enhancing appreciation of the relationship between the mathematics concepts taught and the physics problem they are required to solve. In addition, there are computer technologies that can illustrate some of the problems in practical and visualisable ways (for example, Interactive Physics). Also, working in collaboration with mathematics teachers so that, whenever possible, they can use some of the relevant physics problems in their mathematics classes. These, complemented by the teaching of other skills that enhance meaningful learning, such as metacognitive reflections or questioning such as constructive peer and student-teacher argumentations, will contribute towards successful use of mathematics in physics problem solving tasks and more widely accessible knowledge structures that students can more meaningfully connect across subject areas and disciplines.

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