The Learning & Teaching of Equivalent Fractions for Conceptual Understanding
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Teach Less, Learn More
A call from PM Lee Hsien Loong during his inaugural National Day Rally in 2004 for schools and teachers to:
• teach less
• improve the quality of interaction between teachers and students
• equip students with the knowledge, skills and values to prepare them for life

Singapore’s Mathematical Problem Solving Framework

Educational Achievements
• High level of achievement in Math & Science – top scorer (TIMMS)
• Scored well in international test of reading skills (Netherlands-based International Association of Educational Achievement)
• Performed well in international competitions (Math & Science Olympiads)
So Why the Need for Curriculum Reforms?

- Current state may not serve as well in the future
- 21st century competencies and capacities in response to globalisation and knowledge economies
- CRPP research on classroom practices – classroom pedagogy mainly teacher-centred with high levels of students on task behaviour, reliance on textbooks and worksheets

S’pore Primary Math Syllabus on ‘Fractions’

Equivalent Fractions in the Singapore Textbook

Equivalent Fractions in the Singapore Textbook
Regular Teaching Practice

- Planning and teaching will normally include
  - Teaching a Mathematical skill
  - Developing Mathematical accuracy
  - Developing speed in Mathematical computation
  - Teach application of a mathematical skill into a different context (usually in the form of word problems).

- Student learning is assessed through feedback from assignments, short quizzes, informal tests, topical tests or mental sums.

- Ultimately, the purpose is to enable students to work efficiently in pencil and paper tests/exams where students will need to maximize their time when answering the questions provided.

- This simply means students are taught "how do I do it?" but not "why am I doing it?!".

### Why Conceptual Understanding of Equivalent Fractions (EF)?

Conceptual understanding of EF is essential for learning of advanced topics in Primary 4, Primary 5 and Primary 6.

Pre-test conducted in 2006

<table>
<thead>
<tr>
<th>Pupils have difficulties in:</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpreting fractions as equal parts of a whole.</td>
<td>36.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordering unit fractions</td>
<td>2.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordering of mixed numbers with unlike and unrelated fractional parts - ascending order</td>
<td>45.8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understanding the term “equivalent”; some thought they referred to factors while others thought they referred to like fractions</td>
<td>65.8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtracting a fraction from a mixed fraction and relied on algorithm knowledge and did not use the visual diagram provided in the test</td>
<td>50.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Abundant studies about children’s difficulties in learning fractions (examples: Behr, 1984; Davydov, 1991; Mack, 1990; Kieren, 1993)

- Mathematical rules learnt in Whole Numbers are displaced with a higher level of abstraction in Fractions

As one encounters fractions, the mathematics takes a qualitative leap in sophistication. Suddenly, meanings and models and symbols that worked when adding, subtracting, multiplying, and dividing whole numbers are not as useful. Lamon, Susan J. (2006)

- Hence it is imperative to support the reconstruction of the number system in the child’s mind.
The Study of Equivalent Fractions @ Cedar Primary

Semester 2 Year 2006

15 Teachers
A Facilitator
A Resource Person
2 Research Lessons

Semester 2 Year 2007

3 Teachers
A Facilitator
4 Critical Friends
3 Resource Persons
2 Research Lessons

What constitutes Conceptual Understanding of Equivalent Fractions?

- Interpret and articulate the concept of fractions
- Explain understanding using pictorial representations or personal anecdotes
- Generate and explain the short-cut method of equivalent fractions
- Apply the knowledge in problem solving

Concrete-Pictorial-Abstract Model

MOE Syllabus. With Reference to the Textbook “My Pals are Here”

Pictorial representations is the bridge from Concrete to Abstract. Importance of reiteration of concepts by varying the visuals.

Manipulatives strengthen students’ understanding and conception of mathematical concepts (see for example Bruner, 1975 & Mason, 1996).

Students become independent of pictorial and concrete in computation of algorithms.

Use of manipulative supported by corresponding demonstration of visuals

Capitalizing on pupils’ understanding of visual to allow them to generate mathematical rules and generalizations

Pathway to deep understanding and increase appreciation of value of CPA Model

Collaborative setting of Lesson Study

A concerted effort in using CPA model

Deeper understanding of the model

Manipulatives have to be very well thought through

Important for teacher’s and students manipulatives to be similar in shape and colour

Circles → Rectangles → Other shapes
Mason’s Spiraling Helix Model as a Vehicle in utilisation of CPA model

Mason, 1996

Pictorial
Concrete
Abstract

Articulating
Manipulating
Getting a sense of

Building a TLLM classroom discourse around the CPA –Learning Helix framework

Concrete
Abstract

• Teachers gained a better understanding of how to better support students
construction of knowledge on fractions and reconstruction of their
knowledge of numbers.
• Consequently this induces the reduction of students dependence on
teachers’, as they are encouraged to think for themselves and articulate
their understanding

Building a TLLM classroom discourse around the CPA –Learning Helix framework

• Challenge: Students rarely get the opportunity to articulate their mathematical
thinking in regular lessons.
• Choral reading is a good strategy in building students mathematical
vocabulary and confidence
• Piaget (1981) and von Glaserfeld (1990) emphasised that using language
advances the child’s mental powers. This lends credence to Mason’s spiral
helix.

4 Research Lessons Across 2 Lesson Study Cycles

Research Lesson 1: 15 August 2006
Mr Sean Teng (Novice Teacher) 1y3m teaching experience
Class Profile: ‘Mixed’ Ability

Research Lesson 2: 23 August 2006
Mr Naufal (Novice Teacher) 3m teaching experience
Class Profile: ‘Mixed’ Ability

Research Lesson 3: 27 August 2007
Mr Naufal (Novice Teacher) 1y3m teaching experience
Class Profile: ‘Low’ Ability

Research Lesson 4: 21 September 2007
Mrs Elaine Loke (Experienced Teacher) 8y teaching experience
Class Profile: ‘Mixed’ Ability
### How was the concept of Equivalent Fractions addressed?

#### RL1 in Cycle 2
- **Review of part-whole concept**
- **Teacher dictates folding and shading of rectangular strips.**
- **Class discussion of strips**
- **Introduction of equivalent fractions & algorithm of fraction conversion**

#### RL2 in Cycle 2
- **Review of part-whole concept.**
- **Teacher models writing and reading fractions.**
- **Teacher dictates shading of pre-folded rectangular strips.**
- **Class discussion of strips**
- **Introduction of equivalent fractions & development of conceptual understanding of EF.**

#### RL3 in Cycle 4
- **Review of part-whole concept**
- **Students write shaded fraction.**
- **Class discussion in checking fractions**
- **Given 8 circles with different shaded fraction, students are to identify equivalent fractions from non-equivalent fractions and sort them into 2 groups.**
- **Class discussion. Development of conceptual understanding of EF. Students are encouraged to explain their answers.**
- **Term 'Equivalent' is introduced.**
- **Testing for understanding.**
- **Students encouraged to explain answers.**

#### RL4 in Cycle 4
- **Review of part-whole concept**
- **Given 8 circles with different shaded fractions, students are to identify equivalent fractions from non-equivalent fractions and sort them into 2 groups.**
- **Class discussion. Development of conceptual understanding of EF. Students are encouraged to explain their answers.**
- **Deepen conceptual understanding of EF. Equivalent versus equal?**
- **Choral reading technique**
- **Testing Understanding**
- **Discussion of mathematical shortcuts suggested by students.**
4 Research Lessons Across 2 Lesson Study Cycles

When was the concept of Equivalent Fractions addressed?

- RL1 in Cycle 2
  - EF Concept was addressed
  - 0 Min
- RL2 in Cycle 2
  - EF Concept was addressed
  - 70 Mins
- RL3 in Cycle 4
  - EF Concept was addressed
  - 0 Min
  - 42 Mins
  - 80 Mins
- RL4 in Cycle 4
  - EF Concept was addressed
  - 85 Mins
  - 0 Min
  - 20 Mins

Teachers’ Learning Points

- Draw on students’ prior knowledge of fractions, focused on equivalent fractions
- Teach students to observe, identify and compare the patterns before teaching concept of equivalent fractions
- Represent fractions in concrete terms as well as symbolically
- Capitalize on teachable moments, such as reiteration of the reason why equivalent fractions are represented differently although they are of the same value.

Teachers’ Learning Points

- The recapitulation should have been done in the previous lesson and not during the RL.
- There should be more repetition of the teaching points so choral reading is encouraged.
- Most pupils had difficulty articulating as they are not used to the sharing discourse
- The questioning technique is a vital component of the lesson.

Conclusion

- The research lessons have provided an authentic experience for teachers to develop a systematic pedagogical model of using Concrete-Pictorial-Abstract in teaching a difficult math topic.
- The research lessons are not the best lessons to showcase; yet are powerful tools from which continuous improvements may be pursued.
- Given the difficulty of the topic, it is best to extend the lesson study process to include the whole unit of EF, instead of just focusing on the one lesson on conceptual understanding.
- Lesson study an important teacher professional development tool
Acknowledgements

• Mrs Shirley Ho-Woo, Principal of Cedar Primary
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• Cedar teachers involved in the 2 EF cycles
• WALS organising committee

How would you teach Equivalent Fractions?